Theory of Semiconductor Devices (반도체 소자 이론)

Lecture 5. The Semiconductor in Equilibrium

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Equilibrium Distribution of Electrons and Holes

Equilibrium : No external forces such as voltages, electric fields, magnetic fields, or temperature gradients.

$$n(E) = g_c(E) f_F(E)$$

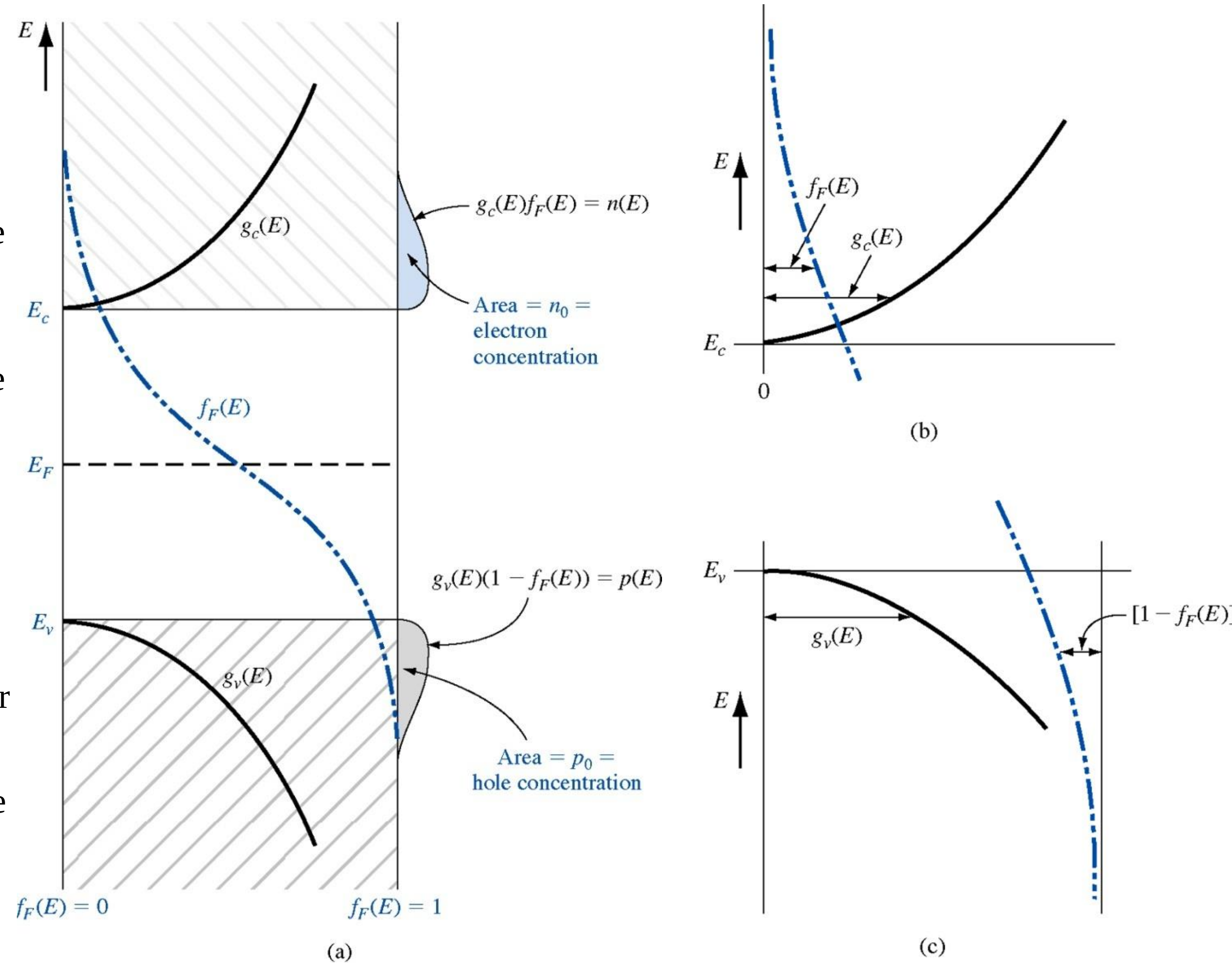
$g_c(E)$: density of states in conduction band per unit volume per unit energy

$n(E)$: density of electrons in conduction band per unit volume per unit energy

$$p(E) = g_v(E)[1 - f_F(E)]$$

$g_v(E)$: density of *empty* states in valence band per unit volume per unit energy

$p(E)$: density of holes in valence band per unit volume per unit energy



Thermal equilibrium concentration of electrons and holes

$$n_0 = \int g_c(E) f_F(E)\, dE$$

$$= \int_{E_c}^{\infty} \frac{4\pi (2m_n^*)^{3/2}}{h^3} \sqrt{E - E_c}\, \exp\left[\frac{-(E - E_F)}{kT}\right] dE$$

$$= \frac{4\pi (2m_n^* kT)^{3/2}}{h^3} \exp\left[\frac{-(E_c - E_F)}{kT}\right] \int_0^{\infty} \eta^{1/2} \exp(-\eta)\, d\eta$$

$$= 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2} \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

$$= N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

$$f_F(E) = \frac{1}{1 + \exp\dfrac{(E - E_F)}{kT}} \approx \exp\frac{[-(E - E_F)]}{kT}$$

$$\eta = \frac{E - E_c}{kT}$$

$$\int_0^{\infty} \eta^{1/2} \exp(-\eta)\, d\eta = \frac{1}{2}\sqrt{\pi}$$

$$N_c = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2}$$

Effective density of states function in the conduction band

$$p_0 = \int g_v(E)[1 - f_F(E)]\, dE = \int_{-\infty}^{E_v} \frac{4\pi (2m_p^*)^{3/2}}{h^3} \sqrt{E_v - E}\, \exp\left[\frac{-(E_F - E)}{kT}\right] dE$$

$$= \frac{-4\pi (2m_p^* kT)^{3/2}}{h^3} \exp\left[\frac{-(E_F - E_v)}{kT}\right] \int_{+\infty}^{0} (\eta')^{1/2} \exp(-\eta')\, d\eta'$$

$$= 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2} \exp\left[\frac{-(E_F - E_v)}{kT}\right] = N_v \exp\left[\frac{-(E_F - E_v)}{kT}\right]$$

$$1 - f_F(E) = \frac{1}{1 + \exp\left(\dfrac{E_F - E}{kT}\right)} \approx \exp\left[\frac{-(E_F - E)}{kT}\right]$$

$$N_v = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2}$$

The Fermi-Dirac integral, changing variables with $\eta \equiv (E - E_C)/kT$ and $\eta_F \equiv (E_F - E_C)/kT$, is given by

$$F_{1/2}\left(\frac{E_F - E_C}{kT}\right) \equiv F_{1/2}(\eta_F) = \int_{E_C}^{\infty} \frac{[(E - E_C)/kT]^{1/2}}{1 + \exp[(E - E_F)/kT]} \frac{dE}{kT}$$

$$= \int_0^{\infty} \frac{\eta^{1/2}}{1 + \exp(\eta - \eta_F)} d\eta \qquad (19)$$

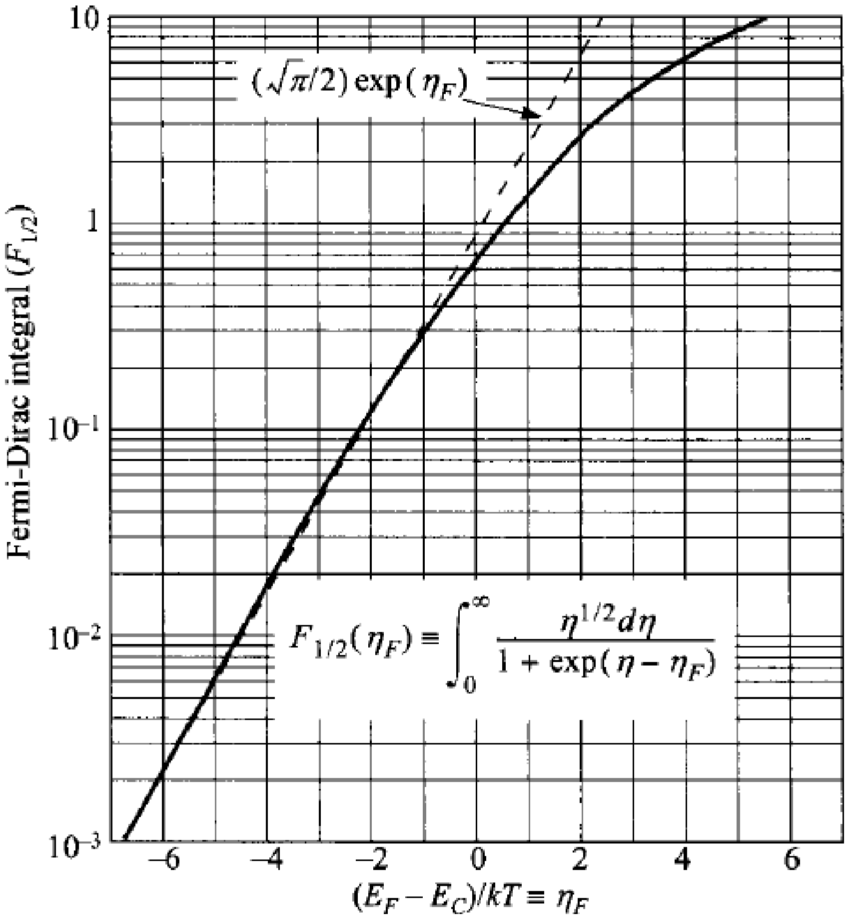


Fig. 8 Fermi-Dirac integral $F_{1/2}$ as a function of Fermi energy. (After Ref. 27.) Dashed line is approximation of Boltzmann statistics.

$$E_F - E_C \approx kT\left[\ln\left(\frac{n}{N_C}\right) + 2^{-3/2}\left(\frac{n}{N_C}\right)\right]$$

$$E_V - E_F \approx kT\left[\ln\left(\frac{p}{N_V}\right) + 2^{-3/2}\left(\frac{p}{N_V}\right)\right]$$

Intrinsic Carrier Concentration : different with Equilibrium Concentration

For an intrinsic semiconductor, the concentration of electrons in the conduction band is equal to the concentration of holes in the valence band.

$$n_0 = n_i = N_c \exp\left[\frac{-(E_c - E_{Fi})}{kT}\right]$$

$$p_0 = p_i = n_i = N_v \exp\left[\frac{-(E_{Fi} - E_v)}{kT}\right]$$

$$n_i^2 = N_c N_v \exp\left[\frac{-(E_c - E_{Fi})}{kT}\right] \cdot \exp\left[\frac{-(E_{Fi} - E_v)}{kT}\right]$$

$$n_i^2 = N_c N_v \exp\left[\frac{-(E_c - E_v)}{kT}\right] = N_c N_v \exp\left[\frac{-E_g}{kT}\right]$$

E_{Fi} : Intrinsic Fermi Energy Level

Table 4.1 | Effective density of states function and density of states effective mass values

	N_c (cm^{-3})	N_v (cm^{-3})	m_n^*/m_0	m_p^*/m_0
Silicon	2.8×10^{19}	1.04×10^{19}	1.08	0.56
Gallium arsenide	4.7×10^{17}	7.0×10^{18}	0.067	0.48
Germanium	1.04×10^{19}	6.0×10^{18}	0.55	0.37

Table 4.2 | Commonly accepted values of n_i at $T = 300$ K

Silicon	$n_i = 1.5 \times 10^{10}$ cm^{-3}
Gallium arsenide	$n_i = 1.8 \times 10^{6}$ cm^{-3}
Germanium	$n_i = 2.4 \times 10^{13}$ cm^{-3}

Calculated n_i value with T =300 K and E_g=1.12 eV is 6.95x10^9 cm^{-3}

: not equal with the normally accepted values of n_i .

- Effective mass and Bandgap energy values are a slight function of temperature
- the density of states function is extracted from 3-D infinite potential well.

Temperature dependence of intrinsic carrier concentration

Intrinsic carrier concentration in GaAs at T = 300 K and 450 K

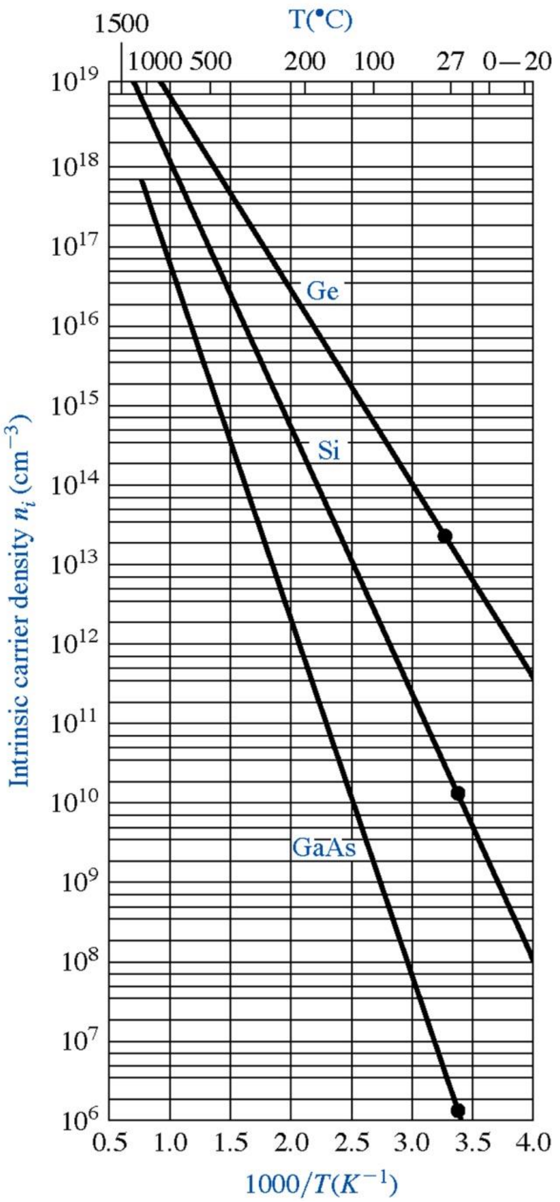
$$n_i^2 = (4.7\times10^{17})(7.0\times10^{18})\exp\left(\frac{-1.42}{0.0259}\right) = 5.09\times10^{12}$$

$$n_i = 2.26\times10^{6}\ \text{cm}^{-3}$$

$$n_i^2 = (4.7\times10^{17})(7.0\times10^{18})\left(\frac{450}{300}\right)^{3/2}\exp\left(\frac{-1.42}{0.03885}\right) = 1.48\times10^{21}$$

$$n_i = 3.85\times10^{10}\ \text{cm}^{-3}$$

4 orders of magnitude difference over 150°C temperature increase !!



Intrinsic Fermi-Level Position

$$N_c\exp\left[\frac{-(E_c-E_{Fi})}{kT}\right] = N_v\exp\left[\frac{-(E_{Fi}-E_v)}{kT}\right]$$

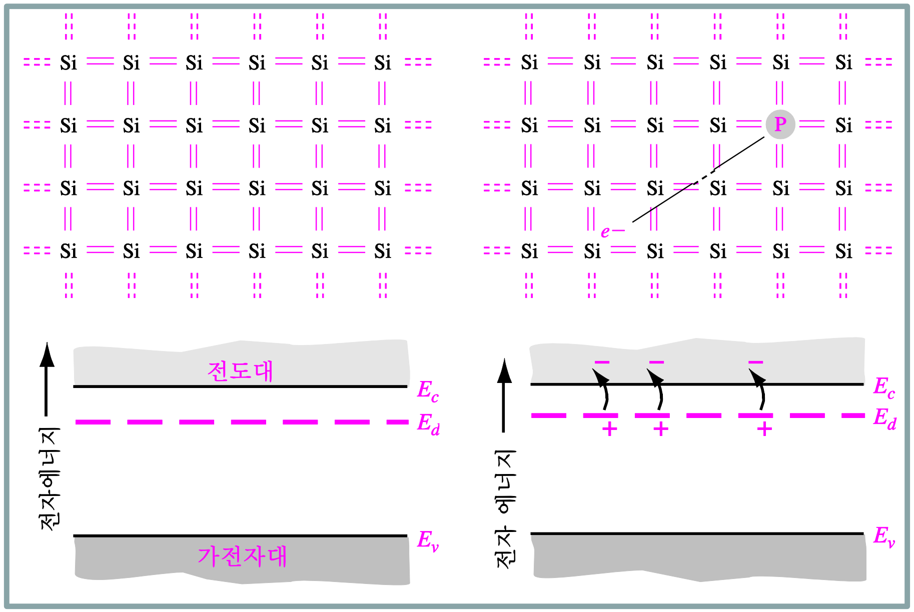
$$E_{Fi} = \frac{1}{2}(E_c+E_v) + \frac{1}{2}kT\ln\left(\frac{N_v}{N_c}\right)$$

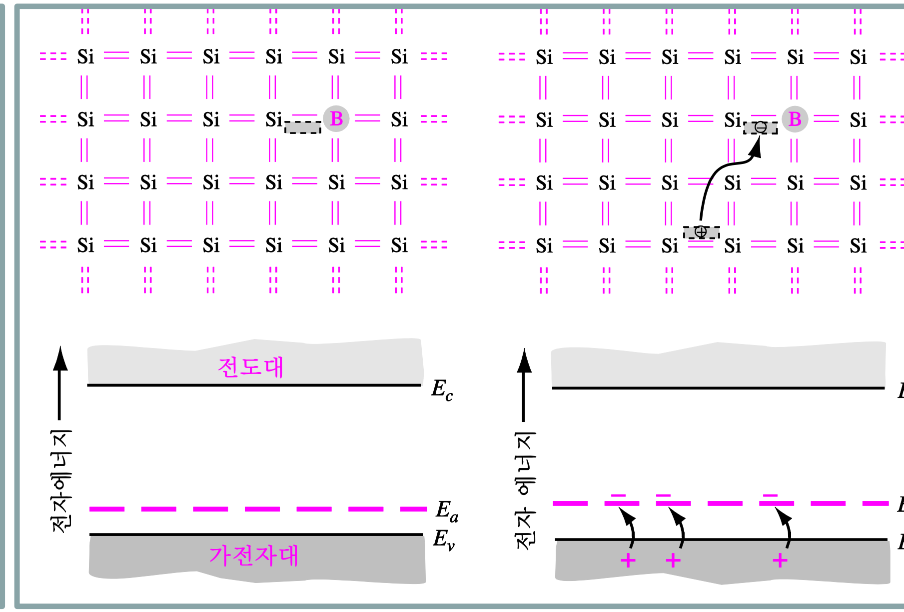
$$E_{Fi} = \frac{1}{2}(E_c+E_v) + \frac{3}{4}kT\ln\left(\frac{m_p^*}{m_n^*}\right) \qquad E_{Fi} - E_{\text{midgap}} = \frac{3}{4}kT\ln\left(\frac{m_p^*}{m_n^*}\right)$$

Doping

Group V element doping N-type semiconductor Donor impurity atom

Group III element doping P-type semiconductor Acceptor impurity atom





Ionization Energy

The energy required to elevate the donor electron into the conduction band.

Impurity	Ionization energy (eV) Si	Ionization energy (eV) Ge
Donors		
Phosphorus	0.045	0.012
Arsenic	0.05	0.0127
Acceptors		
Boron	0.045	0.0104
Aluminum	0.06	0.0102

Impurity	Ionization energy (eV)
Donors	
Selenium	0.0059
Tellurium	0.0058
Silicon	0.0058
Germanium	0.0061
Acceptors	
Beryllium	0.028
Zinc	0.0307
Cadmium	0.0347
Silicon	0.0345
Germanium	0.0404

Impurities for GaAs

In GaAs, if a silicon atom replaces Ga, the Si will act as a donor, but if the Si replaces As atom, then the Si act as an acceptor. → 'amphoteric'

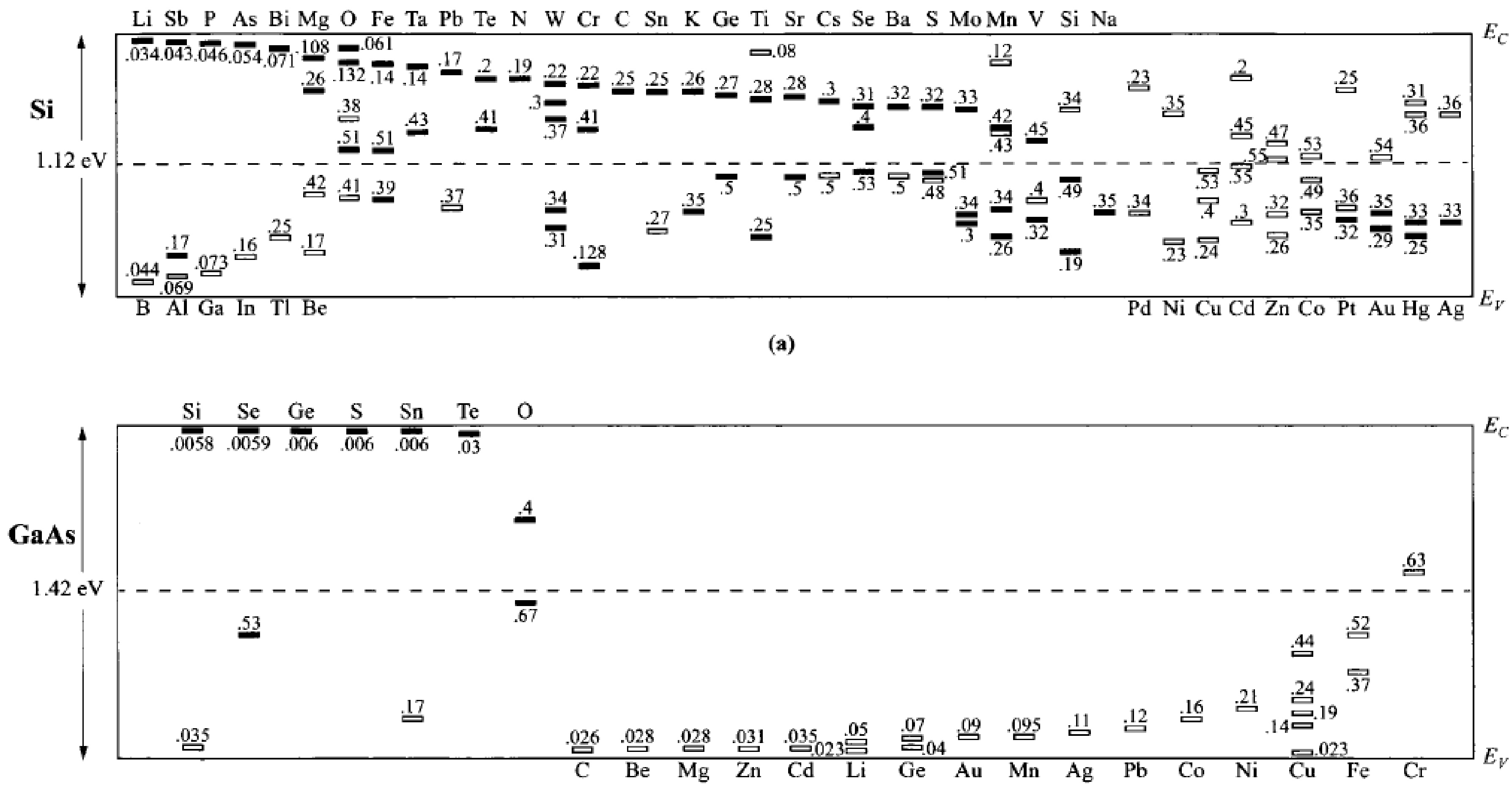


Fig. 10 Measured ionization energies for varies impurities in (a) Si and (b) GaAs. Levels below the gap center are measured from E_V. Levels above the gap center are measured from E_C. Solid bars represent donor levels and hollow boxes represent acceptor levels. (After Refs. 29, 31, 34, and 35.)

Extrinsic Semiconductor

A semiconductor in which controlled amount of specific dopant or impurity atoms have been added so that the thermal equilibrium electron and hole concentrations are different from the intrinsic carrier concentration. One type of carrier will predominate!!

Adding donor or acceptor atoms → invoke change of the distribution of carriers → Fermi energy level should be changed !!

At equilibrium,

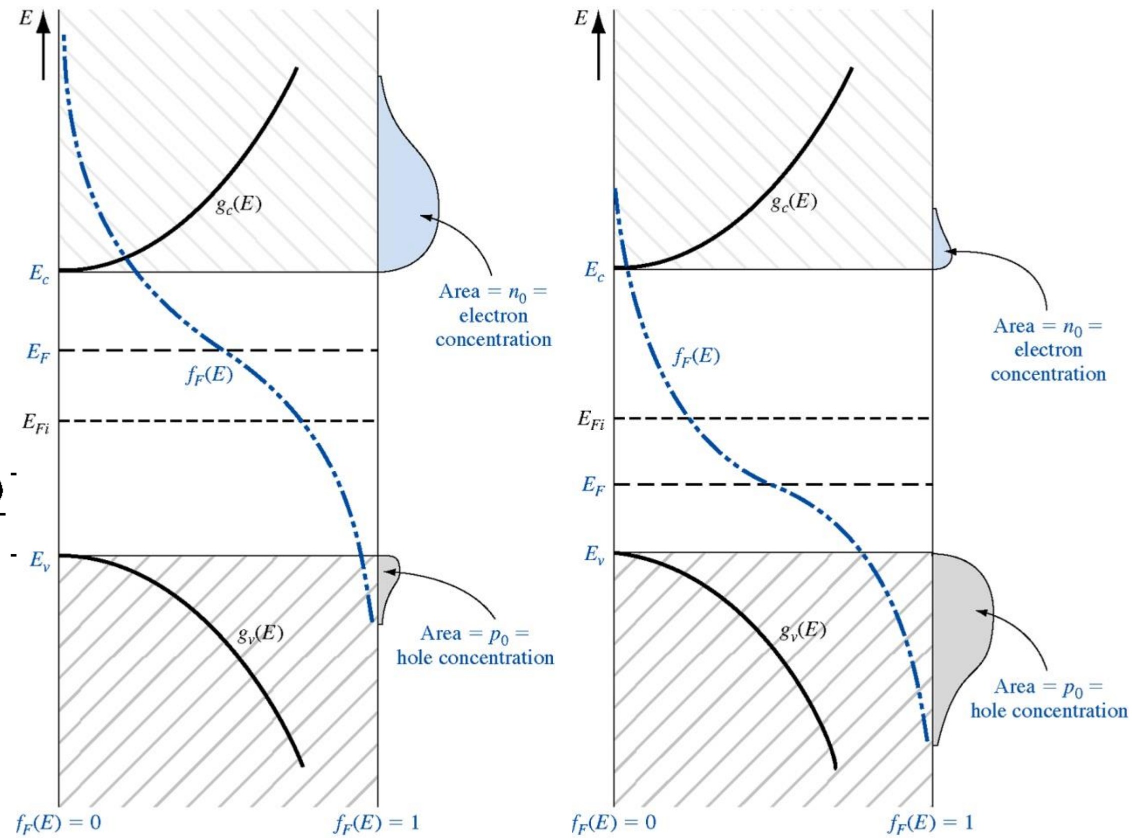
$$n_0 = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

$$= N_c \exp\left[\frac{-(E_c - E_{Fi}) + (E_F - E_{Fi})}{kT}\right]$$

$$= N_c \exp\left[\frac{-(E_c - E_{Fi})}{kT}\right] \exp\left[\frac{(E_F - E_{Fi})}{kT}\right]$$

$$= n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right]$$

$$p_0 = n_i \exp\left[\frac{-(E_F - E_{Fi})}{kT}\right]$$



The n_0p_0 product in equilibrium

$$n_0 p_0 = N_c N_v \exp\left[\frac{-(E_c - E_F)}{kT}\right] \exp\left[\frac{-(E_F - E_v)}{kT}\right] = N_c N_v \exp\left[\frac{-E_g}{kT}\right] = n_i^2$$

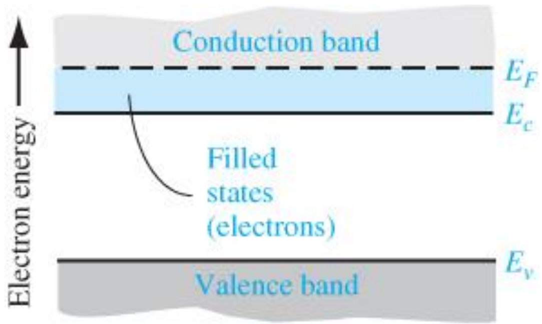
At equilibrium, n_0p_0 product is same regardless of the impurity concentrations.

Keep in mind that the above equation is based on the Maxwell-Boltzmann approximation.

Degenerate Semiconductors

Nondegenerate n-type semiconductors :

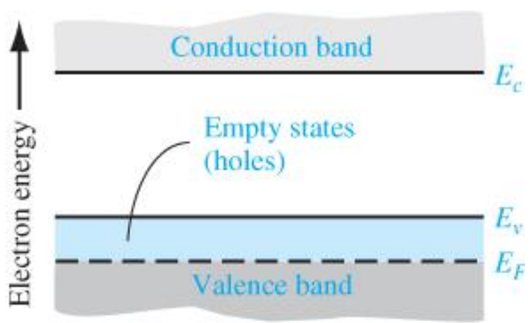
Impurity concentration is small compared to the density of semiconductor atoms. → No interactions between the impurity atoms and between donor electrons. → Discrete and noninteracting donor energy states.



Degenerate n-type semiconductors :

Impurity concentration is increased and donor electrons interact with each other. → Discrete donor energy will split into a band of energies. → The band of donor states widens and overlap the bottom of the conduction band.

This overlap occurs when the donor concentration becomes comparable with the effective density of states, N_c .



Probability Function

$$n_d = \frac{N_d}{1 + \frac{1}{2}\exp\left(\frac{E_d - E_F}{kT}\right)}$$

$$n_d = N_d - N_d^+$$

N_d : the density of donor atoms

n_d : the density of electrons occupying donor level

E_d : the energy of donor level

N_d^+ : the concentration of ionized donors

Similarly, $$p_a = \frac{N_a}{1 + \frac{1}{g}\exp\left(\frac{E_F - E_a}{kT}\right)} = N_a - N_a^-$$

N_a : the density of acceptor atoms

n_a : the density of holes in the acceptor states

E_a : the energy of acceptor level

N_a^- : the concentration of ionized acceptors

For $(E_d - E_F) \gg kT$

$$n_d \approx \frac{N_d}{\frac{1}{2}\exp\left(\frac{E_d - E_F}{kT}\right)} = 2N_d \exp\left[\frac{-(E_d - E_F)}{kT}\right]$$

$$\frac{n_d}{n_d + n_0} = \frac{2N_d \exp\left[\frac{-(E_d - E_F)}{kT}\right]}{2N_d \exp\left[\frac{-(E_d - E_F)}{kT}\right] + N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right]} = \frac{1}{1 + \frac{N_c}{2N_d}\exp\left[\frac{-(E_c - E_d)}{kT}\right]}$$

: Unionized fraction

Example 4.7 : Consider phosphorus doping in silicon , for T = 300K with $N_d = 10^{16}$ cm^{-3}

$$\frac{n_d}{n_0 + n_d} = \frac{1}{1 + \dfrac{2.8 \times 10^{19}}{2(10^{16})} \exp\left(\dfrac{-0.045}{0.0259}\right)} = 0.0041 = 0.41\%$$

There are very few electrons in the donor state, 0.41 %, almost completely ionized!!

Complete ionization

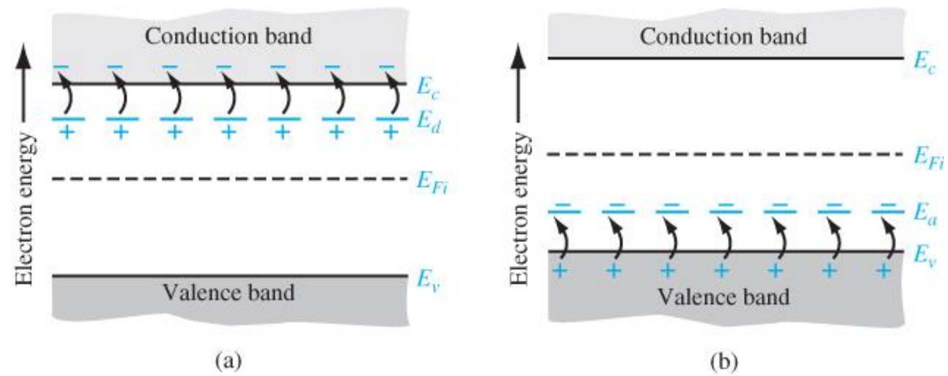


Figure 4.12 | Energy-band diagrams showing complete ionization of (a) donor states and (b) acceptor states.

Freeze-out at T = 0 K

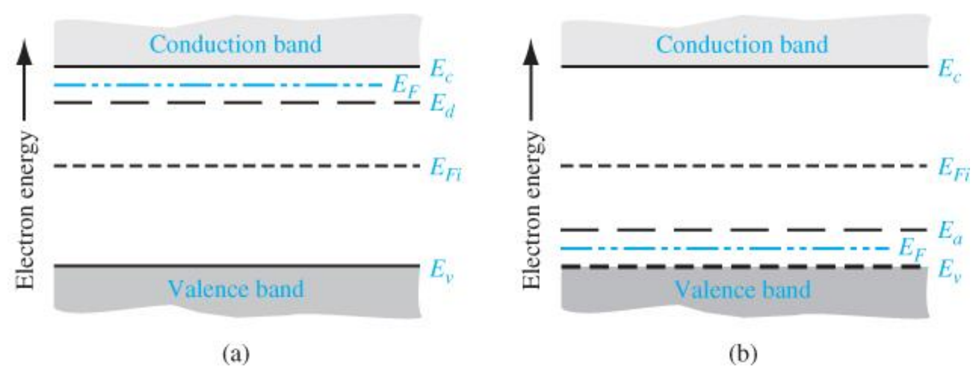


Figure 4.13 | Energy-band diagram at T = 0 K for (a) n-type and (b) p-type semiconductors.

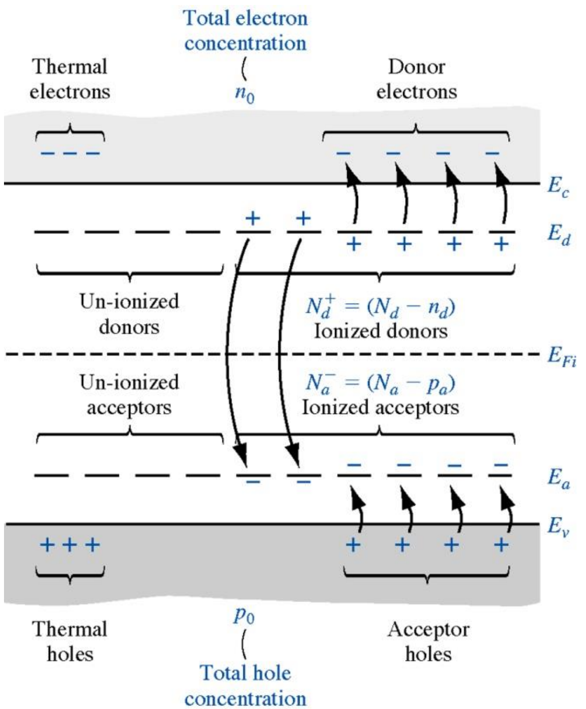
Compensated Semiconductor

: Both donor and acceptor impurity atoms in the same region

$N_d > N_a$: n-type semiconductor

$N_a > N_d$: p-type semiconductor

$N_a = N_d$: intrinsic semiconductor



Charge Neutrality

$$n_0 + N_a^- = p_0 + N_d^+ \quad \text{or} \quad n_0 + (N_a - p_a) = p_0 + (N_d - n_d)$$

For complete ionization :

$$n_0 + N_a = p_0 + N_d \Rightarrow n_0 + N_a = \frac{n_i^2}{n_0} + N_d$$

$$\Rightarrow n_0^2 - (N_d - N_a)n_0 - n_i^2 = 0$$

$$\Rightarrow n_0 = \frac{(N_d - N_a)}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2}$$

$$p_0 = \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2}$$

Example 4.9 : Consider an n-type silicon at T = 300K with $N_d = 10^{16}$ cm^{-3} and $N_a = 0$. ni = 1.5x10^{15} cm^{-3}

$$n_0 = \frac{10^{16}}{2} + \sqrt{\left(\frac{10^{16}}{2}\right)^2 + (1.5 \times 10^{10})^2} \approx 10^{16} \text{ cm}^{-3}$$

$$p_0 = \frac{n_i^2}{n_0} = \frac{(1.5 \times 10^{10})^2}{1 \times 10^{16}} = 2.25 \times 10^4 \text{ cm}^{-3}$$

For $N_d \gg n_i$, $n_0 = N_d$

Example 4.10 : Consider a germanium at T = 300K with $N_d = 5\times10^{13}$ cm^{-3} and $N_a = 0$. ni = 2.4×10^{13} cm^{-3}

$$n_0 = \frac{5\times10^{13}}{2} + \sqrt{\left(\frac{5\times10^{13}}{2}\right)^2 + (2.4\times10^{13})^2} = 5.97\times10^{13}\ \text{cm}^{-3}$$

$$p_0 = \frac{n_i^2}{n_0} = \frac{(2.4\times10^{13})^2}{5.97\times10^{13}} = 9.65\times10^{12}\ \text{cm}^{-3}$$

For N_d is comparable to n_i, n_0 is influenced by the intrinsic concentration.

Redistribution of carriers

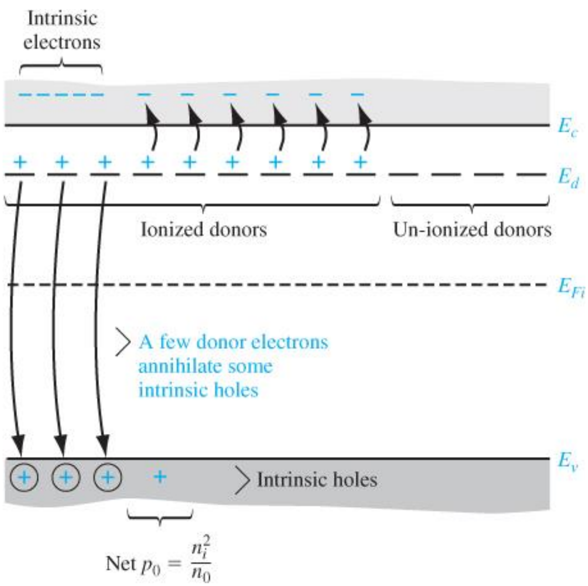


Figure 4.15 | Energy-band diagram showing the redistribution of electrons when donors are added.

n_i is a very strong function of temperature, additional electron-hole pairs are thermally generated as the temperature increases.

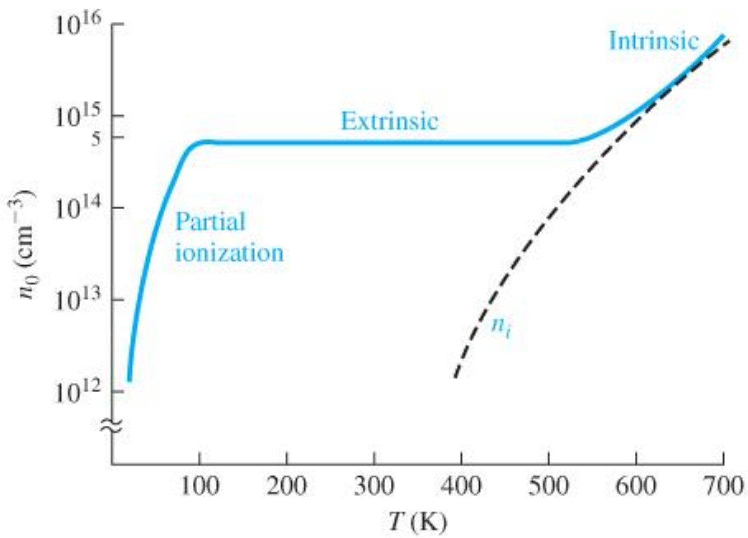


Figure 4.16 | Electron concentration versus temperature showing the three regions: partial ionization, extrinsic, and intrinsic.

Mathematical Derivation

$$n_0 = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right] \Rightarrow E_c - E_F = kT \ln\left(\frac{N_c}{n_0}\right) \Rightarrow E_c - E_F = kT \ln\left(\frac{N_c}{N_d}\right)$$

For $N_d \gg n_i$

$$n_0 = n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right] \Rightarrow E_F - E_{Fi} = kT \ln\left(\frac{n_0}{n_i}\right)$$

$$p_0 = N_v \exp\left[\frac{-(E_F - E_v)}{kT}\right] \Rightarrow E_F - E_v = kT \ln\left(\frac{N_v}{p_0}\right) \Rightarrow E_F - E_v = kT \ln\left(\frac{N_v}{N_a}\right)$$

$$p_0 = n_i \exp\left[\frac{-(E_F - E_{Fi})}{kT}\right] \Rightarrow E_{Fi} - E_F = kT \ln\left(\frac{p_0}{n_i}\right)$$

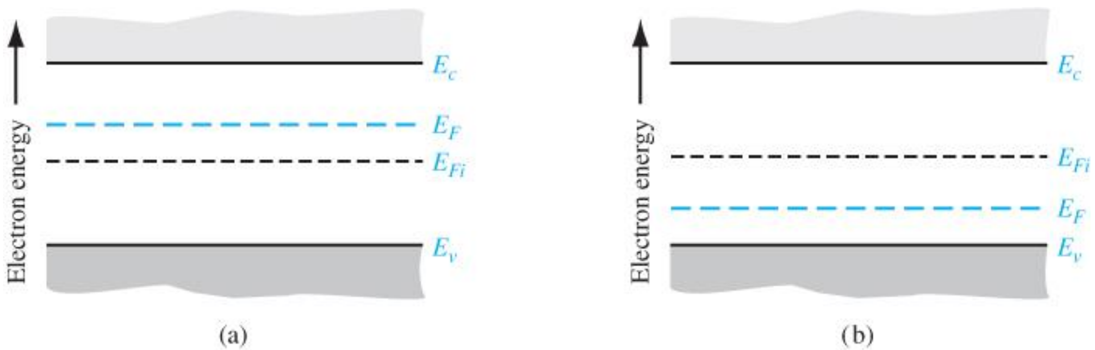


Figure 4.17 | Position of Fermi level for an (a) n-type ($N_d > N_a$) and (b) p-type ($N_a > N_d$) semiconductor.

Variation of E_F with Doping Concentration and Temperature

$$E_F - E_{Fi} = kT \ln\left(\frac{n_0}{n_i}\right)$$

$$E_{Fi} - E_F = kT \ln\left(\frac{p_0}{n_i}\right)$$

$$n_i^2 = N_c N_v \exp\left[\frac{-(E_c - E_v)}{kT}\right] = N_c N_v \exp\left[\frac{-E_g}{kT}\right]$$

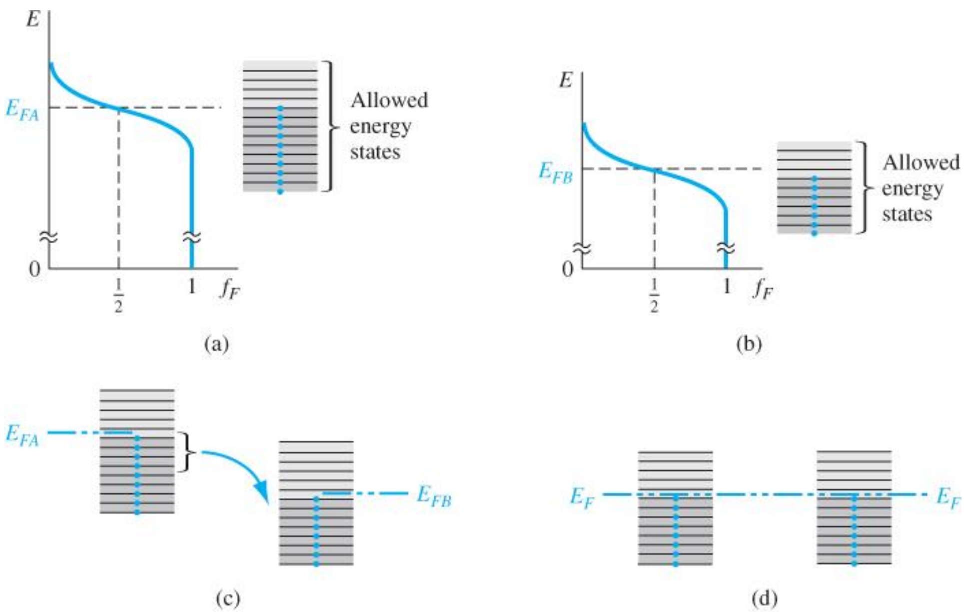


Figure 4.20 | The Fermi energy of (a) material A in thermal equilibrium, (b) material B in thermal equilibrium, (c) materials A and B at the instant they are placed in contact, and (d) materials A and B in contact at thermal equilibrium.