Theory of Semiconductor Devices (반도체 소자 이론)

Lecture 6. Carrier transport phenomena

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An electric field applied to a semiconductor will produce a force on electrons and holes so that they will experience a net acceleration and net movement, provided there are available energy states in the conduction and valence bands. This net movement of charge due to an electric field is called ***drift***. The net drift of charge gives rise to a ***drift current***.

Drift Current Density

Drift current : the net drift of charges due to the electric field

$$J_{drf} = \rho \upsilon_d$$

ρ : charge density

υ_d : drift velocity

For holes,

$$J_{p|drf} = (ep)\upsilon_{dp}$$

υ_{dp} : drift velocity of hole

$$\upsilon_{dp} = \mu_p \mathrm{E}$$

μ_p : hole mobility

Mobility : determine how well the charged particle will move due to an electric field

$$F = m_p^* a = e\mathrm{E} \;\Rightarrow\; J_{p|drf} = (ep)\upsilon_{dp} = e\mu_p p\mathrm{E}$$

Similarly, for electrons

$$\upsilon_{dn} = -\mu_n \mathrm{E} \;\Rightarrow\; J_{n|drf} = \rho\upsilon_{dn} = (-en)\upsilon_{dn}$$

$$\Rightarrow\; J_{n|drf} = (-en)(-\mu_n \mathrm{E}) = e\mu_n n\mathrm{E}$$

Total drift current density :

$$J_{drf} = e(\mu_n n + \mu_p p)\mathrm{E}$$

Table 5.1 | Typical mobility values at $T = 300$ K and low doping concentrations

	μ_n (cm²/V-s)	μ_p (cm²/V-s)
Silicon	1350	480
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Germanium	3900	1900

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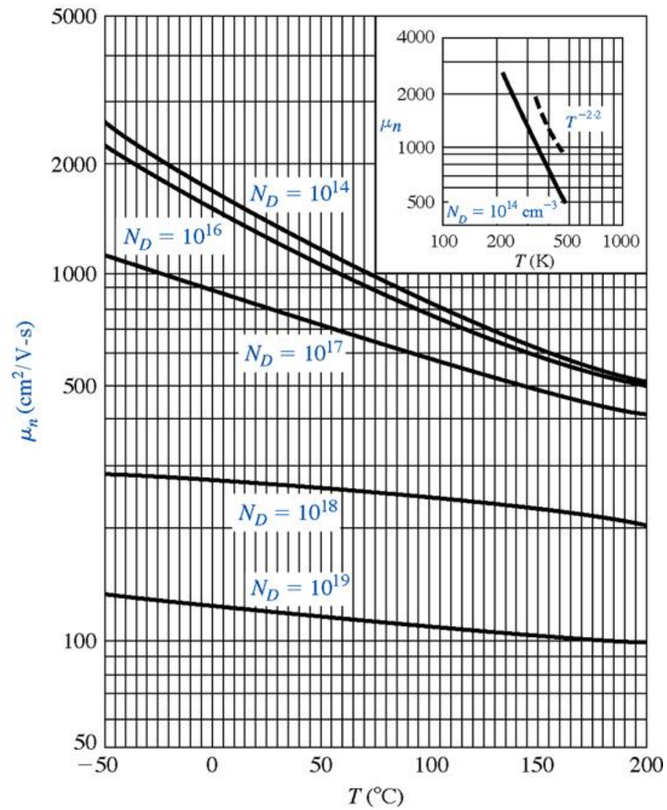
Mobility Effects

$$F = m_p^* \frac{dv}{dt} = e\mathrm{E} \qquad v = \frac{e\mathrm{E}t}{m_p^*}$$

τ_{cp} , τ_{cn} : mean time between collisions

$$\Rightarrow \mu_p = \frac{v_{dp}}{\mathrm{E}} = \frac{e\tau_{cp}}{m_p^*} \qquad \mu_n = \frac{e\tau_{cn}}{m_n^*}$$



Lattice scattering(Phonon scattering) :

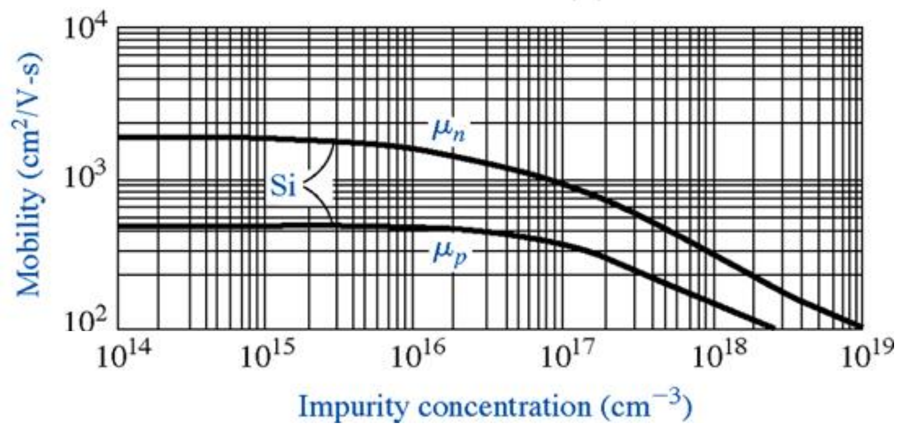
Lattice vibration → disruption in perfect periodic potential → interaction between electrons/holes and lattice atoms

$$\mu_L \propto T^{-3/2}$$

Impurity scattering :

Coulomb interaction between electrons/holes and ionized impurity atoms

$$\mu_I \propto \frac{T^{+3/2}}{N_I}$$



dt/τ : probability of scattering event during the differential time dt

$$\frac{dt}{\tau} = \frac{dt}{\tau_I} + \frac{dt}{\tau_L} \quad \Rightarrow \quad \frac{1}{\mu} = \frac{1}{\mu_I} + \frac{1}{\mu_L}$$

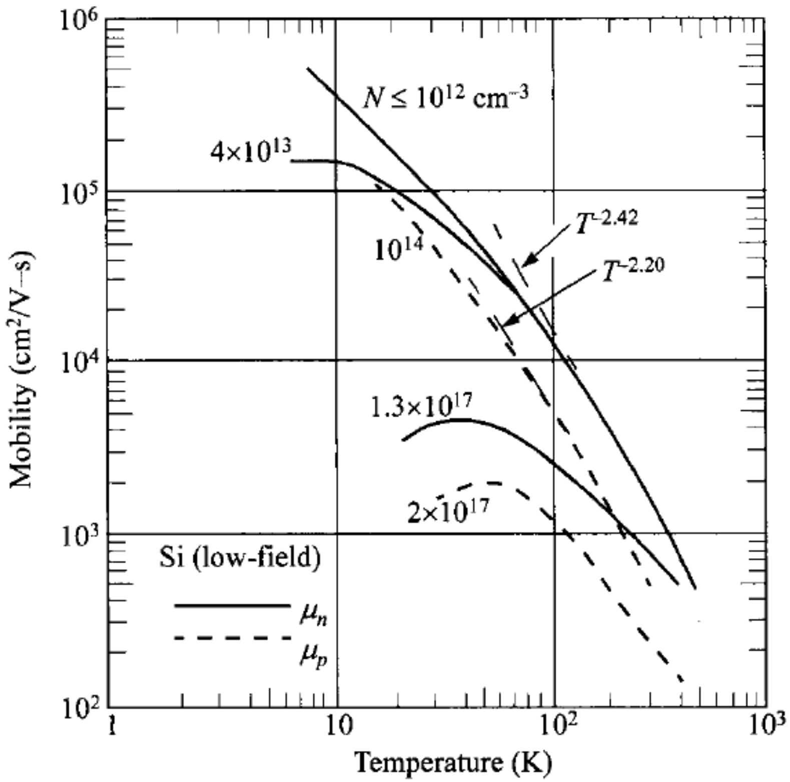


Fig. 16 Mobility of electrons and holes in Si as a function of temperature. (After Ref. 41.)

The measured slopes are different from -3/2 because of other scattering mechanisms. For these pure materials, near room temperature, the mobility varies as $T^{-2.42}$ and $T^{-2.20}$ for n and p-type Si, respectively.

(Sze 3rd edition)

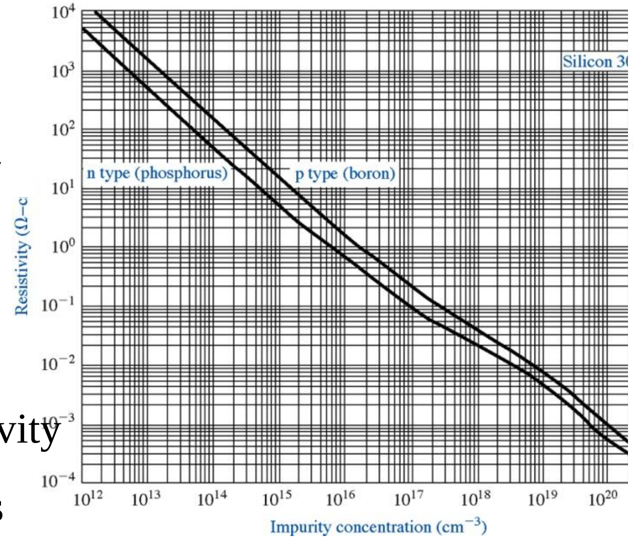
Conductivity

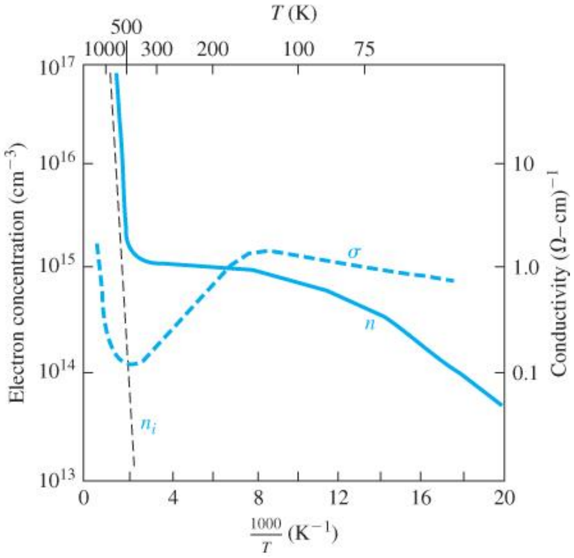
$$J_{drf} = e(\mu_n n + \mu_p p)\mathrm{E} = \sigma \mathrm{E}$$

$$\rho = \frac{1}{\sigma} = \frac{1}{e(\mu_n n + \mu_p p)}$$

Resistivity : Reciprocal of conductivity

depends on the carrier concentrations and mobilities





$$J = \frac{I}{A} \qquad \mathrm{E} = \frac{V}{L} \qquad \frac{I}{A} = \sigma\left(\frac{V}{L}\right)$$

$$\Rightarrow \quad V = \left(\frac{L}{\sigma A}\right) I = \left(\frac{\rho L}{A}\right) I = IR$$

For p-type, $\sigma = e(\mu_n n + \mu_p p) \approx e\mu_p p$

$$\Rightarrow \quad \sigma \approx e\mu_\rho N_a \approx \frac{1}{\rho}$$

: primarily depends on the majority carrier

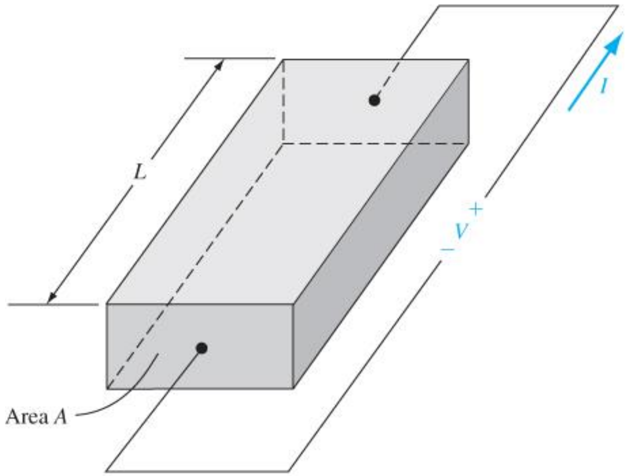


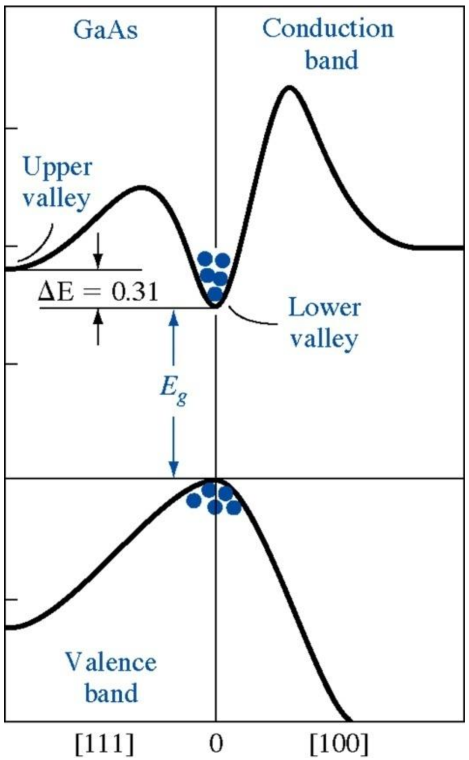
Figure 5.5 | Bar of semiconductor material as a resistor.

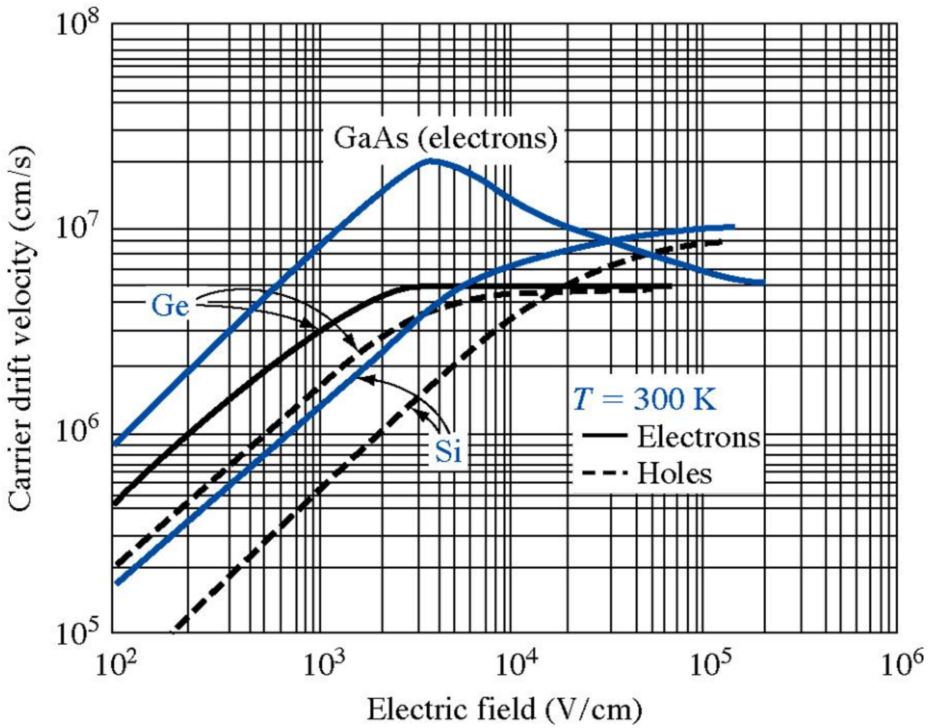
Velocity saturation

At low field :

$$\upsilon_d = \mu \cdot E$$ μ is constant

At high field, velocity is saturated → the current density is also saturated.





For GaAs, the mobility is higher than silicon.

At low field, the situation is same as the case of silicon.

As the field increases, the differential mobility is negative!! → negative differential resistance..

At lower valley, the effective mass is $0.067m_0$, and the effective mass is $0.55m_0$ at the upper valley. (Small effective mass leads to large mobility)

The intervalley transfer at high field, results in the decreasing average drift velocity of electron

Diffusion Current Density

Diffusion : Flow of particles from a region of high concentration toward a region of low concentration.

$$J_{nx|dif} = eD_n \frac{dn}{dx}$$

D_n : electron diffusion coefficient

$$J_{px|dif} = -eD_p \frac{dp}{dx}$$

D_p : hole diffusion coefficient

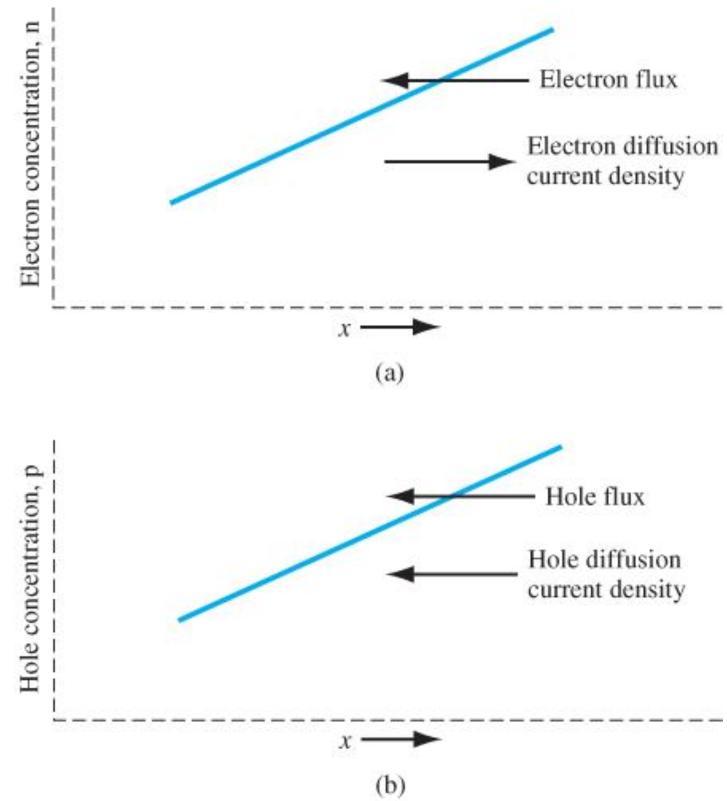


Figure 5.11 | (a) Diffusion of electrons due to a density gradient. (b) Diffusion of holes due to a density gradient.

Total Current Density

$$J = en\mu_n \mathrm{E}_x + ep\mu_p \mathrm{E}_x + eD_n \frac{dn}{dx} - eD_p \frac{dp}{dx}$$

$$J = en\mu_n \mathrm{E} + ep\mu_p \mathrm{E} + eD_n \nabla n - eD_p \nabla p$$

Induced Electric Field

Nonuniformly doped n-type semiconductor in equilibrium

→Fermi energy level is constant

→ Diffusion of majority carriers

→ Separation of positive and negative charge induces an electric field in a direction to oppose the diffusion process

→ The induced electric field prevents any further separation of charge (the mobile carrier concentration is not exactly equal to the fixed impurity concentration)

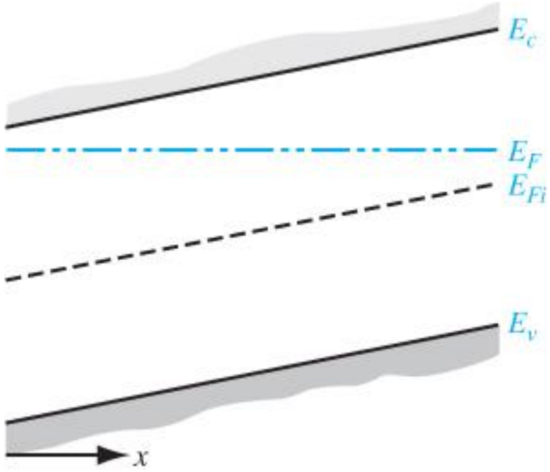


Figure 5.12 | Energy-band diagram for a semiconductor in thermal equilibrium with a nonuniform donor impurity concentration.

$$\phi = +\frac{1}{e}(E_F - E_{Fi}) \qquad \mathrm{E}_x = -\frac{d\phi}{dx} = \frac{1}{e}\frac{dE_{Fi}}{dx}$$

ϕ : electric potential

E_x : electric field in one dimension

$$n_0 = n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right] \approx N_d(x) \qquad E_F - E_{Fi} = kT \ln\left(\frac{N_d(x)}{n_i}\right) \qquad -\frac{dE_{Fi}}{dx} = \frac{kT}{N_d(x)}\frac{dN_d(x)}{dx}$$

$$\Longrightarrow \quad \mathrm{E}_x = -\left(\frac{kT}{e}\right)\frac{1}{N_d(x)}\frac{dN_d(x)}{dx}$$

Induced electric field due to the nonuniform doping concentration.

Einstein Relation

Nonuniformly doped semiconductor in equilibrium. (no external forces)

→ No electron and hole current !!

$$J_n = 0 = en\mu_n \mathrm{E}_x + eD_n \frac{dn}{dx} \qquad J_n = 0 = e\mu_n N_d(x)\mathrm{E}_x + eD_n \frac{dN_d(x)}{dx}$$

$$0 = -e\mu_n N_d(x)\left(\frac{kT}{e}\right)\frac{1}{N_d(x)}\frac{dN_d(x)}{dx} + eD_n\frac{dN_d(x)}{dx} \qquad \mathrm{E}_x = -\left(\frac{kT}{e}\right)\frac{1}{N_d(x)}\frac{dN_d(x)}{dx}$$

$$\Rightarrow \frac{D_n}{\mu_n} = \frac{kT}{e} \qquad \frac{D_p}{\mu_p} = \frac{kT}{e} \qquad \frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{kT}{e}$$

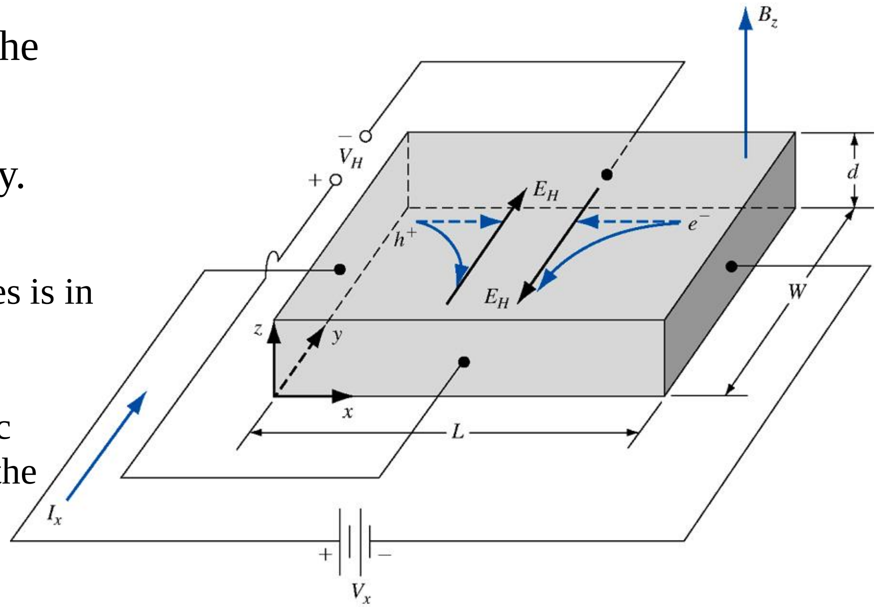
The diffusion coefficient and mobility are not independent parameters!!

Table 5.2 | Typical mobility and diffusion coefficient values at $T = 300$ K ($\mu = \mathrm{cm}^2/\mathrm{V\text{-}s}$ and $D = \mathrm{cm}^2/\mathrm{s}$)

	μ_n	D_n	μ_p	D_p
Silicon	1350	35	480	12.4
Gallium arsenide	8500	220	400	10.4
Germanium	3900	101	1900	49.2

The Hall effect is used to distinguish whether a semiconductor is n-type or p-type and to measure the majority carrier concentration and mobility.

Magnetic probe : measure the magnetic flux density.



$$F = qv \times B$$

The force on both electrons and holes is in the (-y) direction

The buildup of positive/negative charge induces an electric field (*Hall field*) in (+y) direction to exactly balance out the force due to the magnetic field.

$$F = q[\mathrm{E} + v \times B] = 0 \Rightarrow q\mathrm{E}_y = qv_x B_z$$

Hall Voltage : $V_H = +\mathrm{E}_H W$ — E_H is positive in +y direction $\Rightarrow V_H = v_x W B_z$

$V_H > 0 \rightarrow$ p-type

$V_H < 0 \rightarrow$ n-type

For p-type, $v_{dx} = \frac{J_x}{ep} = \frac{I_x}{(ep)(Wd)} \Rightarrow V_H = \frac{I_x B_z}{epd} \Rightarrow p = \frac{I_x B_z}{edV_H}$

For n-type, $V_H = -\frac{I_x B_z}{ned} \Rightarrow n = -\frac{I_x B_z}{edV_H}$

Carrier concentration is determined from current, B-field and Hall voltage.

For mobility measure, $J_x = ep\mu_p \mathrm{E}_x \Rightarrow \frac{I_x}{Wd} = \frac{ep\mu_p V_x}{L} \Rightarrow \mu_p = \frac{I_x L}{epV_x Wd}$ $\quad \mu_n = \frac{I_x L}{enV_x Wd}$

Hall voltage can be measured externally and is given by

$$V_H = \mathscr{E}_y W = \frac{J_x \mathscr{B}_z W}{qp}$$

When scattering is taken into account, the Hall voltage becomes

$$V_H = R_H J_x \mathscr{B}_z W$$

Where R_H is the Hall coefficient and is given by

$$R_H = \frac{r_H}{qp} \qquad p \gg n,$$

$$R_H = -\frac{r_H}{qn} \qquad n \gg p,$$

With a Hall factor $r_H \equiv \frac{\langle \tau_m^2 \rangle}{\langle \tau_m \rangle^2}$

The parameter τ_m for the Hall factor is the mean free time between carrier collisions, which depends on the carrier energy.

r_H = 1.18 for phonon scattering and r_H = 1.93 for ionized-impurity scattering.

In general, r_H lies in the range of 1~2. At very high magnetic fields, it approaches a value slightly below unity.

(Sze 3rd edition)

Hall Measurement