Theory of Semiconductor Devices (반도체 소자 이론)

Lecture 7. Nonequilibrium excess carriers in semiconductors

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In Equilibrium

Generation : the process whereby electrons and holes are created

Recombination : the process whereby electrons and holes are annihilated

Any deviation from thermal equilibrium (temp. change or external excitation)

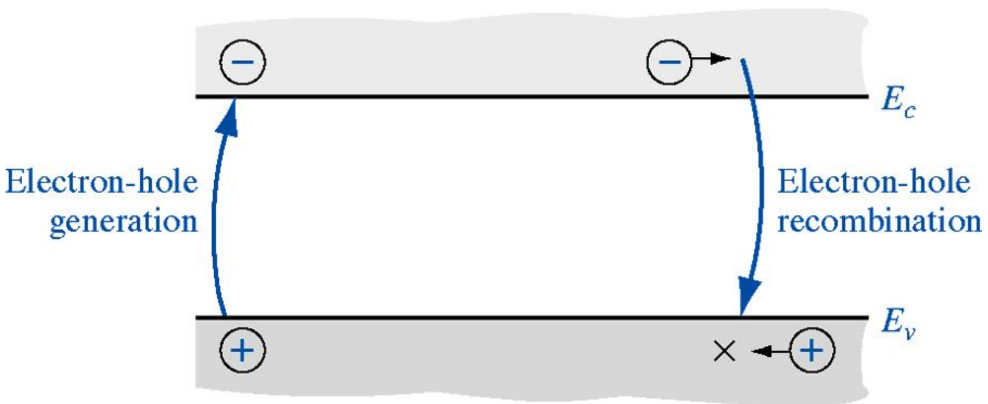
→ Change electrons and hole concentrations

→ New equilibrium electron and hole concentration

$$G_{n0} = G_{p0} \qquad (\#/\text{cm}^3\text{-s})$$

$$R_{n0} = R_{p0}$$

$$G_{n0} = G_{p0} = R_{n0} = R_{p0}$$



Direct Band-to-Band Generation and Recombination

G_{n0} : thermal generation rates of electrons

G_{p0} : thermal generation rates of electrons

R_{n0} : recombination rates of electrons

R_{p0} : recombination rates of holes

Excess Carrier Generation and Recombination

For the direct band-to-band generation,

$$g'_n = g'_p$$

When excess electrons and holes are created,

$$n = n_0 + \delta n$$

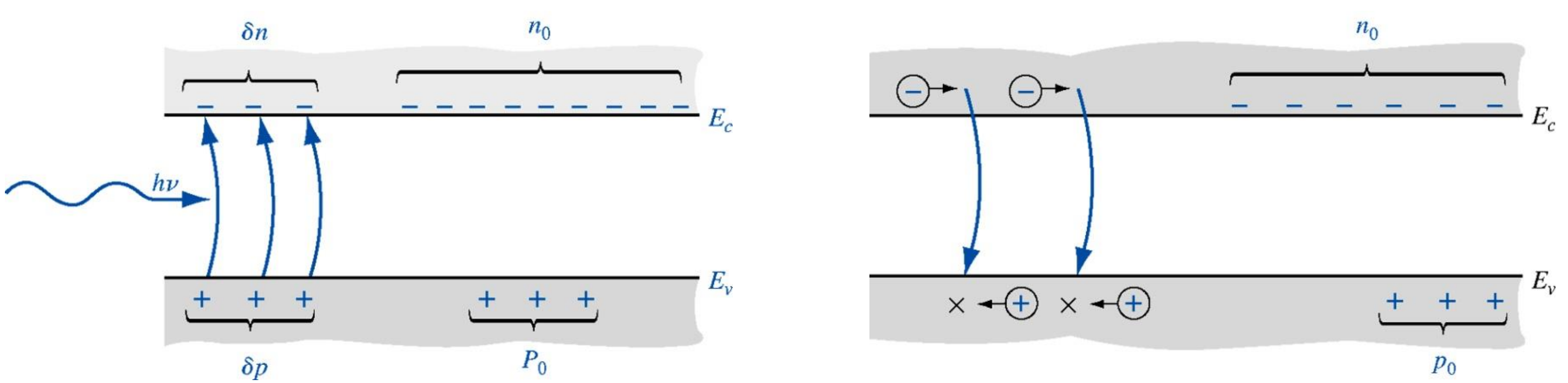
$$p = p_0 + \delta p$$

For spontaneous recombination,

$$R'_n = R'_p$$

Table 6.1 Relevant notations used in Chapter 6

n_0, p_0	Thermal equilibrium electron and hole concentrations (independent of time and also usually positions).
n, p	Total electron and hole concentrations (may be functions of time and/or position).
$\delta n = n - n_0$ $\delta p = p - p_0$	Excess electron and hole concentrations (may be functions of time and/or positions).
g'_n, g'_p	Excess electron and hole generation rates.
R'_n, R'_p	Excess electron and hole recombination rates.
τ_{n0}, τ_{p0}	Excess minority carrier electron and hole lifetimes.



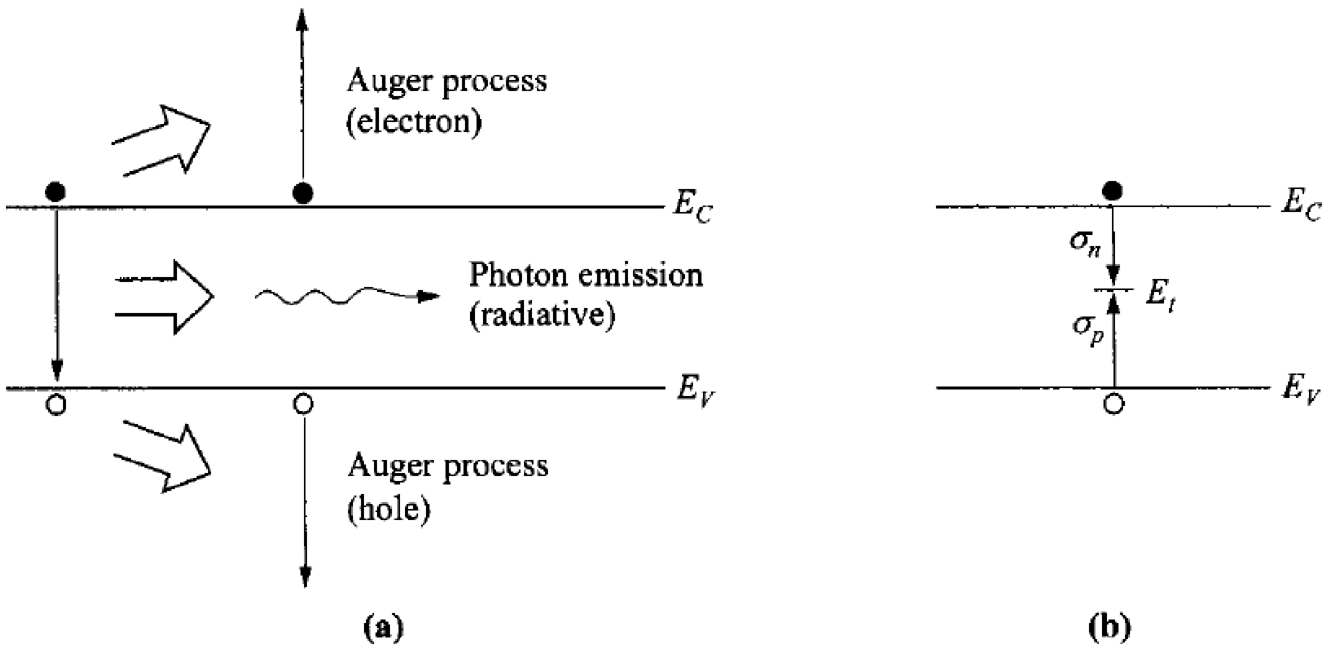


Fig. 25 Recombination processes (the reverse are generation processes). (a) Band-to-band recombination. Energy is exchanged to a radiative or Auger process. (b) Recombination through single-level traps (nonradiative).

The energy of an electron in transition from the conduction band to the valence band is conserved by 1) emission of a photon (radiative process) or 2) by transfer of the energy to another free electron or hole (Auger process). The former process is the inversion of direct optical absoption, and the latter is the inversion of impact ionization.

In indirect-bandgap semiconductors, the dominant transitions are indirect recombination/generation via bulk traps, of density N_t and energy E_t present within the bandgap. The single level recombination can be described by two processes – electron capture and hole capture. The net transition rate can be described by the Shockley-Read-Hall statistics.

Excess Carrier Generation and Recombination

The net rate of change in the electron concentration :

$$\frac{dn(t)}{dt} = \alpha_r\left[n_i^2 - n(t)p(t)\right]$$

$$n(t) = n_0 + \delta n(t)$$

$$p(t) = p_0 + \delta p(t)$$

$$\frac{dn(t)}{dt} = \frac{d(\delta n(t))}{dt} = \alpha_r\left[n_i^2 - (n_0 + \delta n(t))(p_0 + \delta p(t))\right]$$

$$= -\alpha_r \delta n(t)[(n_0 + p_0) + \delta n(t)]$$

($\delta n(t) = \delta p(t)$)

For low-level injection, the magnitude of the excess carrier concentration is small compared with the thermal equilibrium majority carrier concentrations,

For a p-type material ($p_0 \gg n_0$) and $\delta n(t) \ll p_0$,

$$\frac{d(\delta n(t))}{dt} = -\alpha_r p_0 \delta n(t)$$

$\tau_{n0} = (\alpha_r p_0)^{-1}$: Excess minority carrier lifetime

$$\Rightarrow \quad \delta n(t) = \delta n(0)e^{-\alpha_r p_0 t} = \delta n(0)e^{-t/\tau_{n0}}$$

Recombination rate (positive quantity) :

$$R_n' = \frac{-d(\delta n(t))}{dt} = +\alpha_r p_0 \delta n(t) = \frac{\delta n(t)}{\tau_{n0}}$$

$$R_n' = R_p' = \frac{\delta n(t)}{\tau_{n0}}$$

(In case of direct band-to-band recombination)

Similarly, for a n-type material ($n_0 \gg p_0$) and $\delta p(t) \ll n_0$,

$$R_n' = R_p' = \frac{\delta n(t)}{\tau_{p0}}$$

$\tau_{p0} = (\alpha_r n_0)^{-1}$

Continuity Equations

$$F_{px}^{+}(x+dx) = F_{px}^{+}(x) + \frac{\partial F_{px}^{+}}{\partial x} \cdot dx$$

F_{px}^{+} : hole flux (#/cm^2-s)

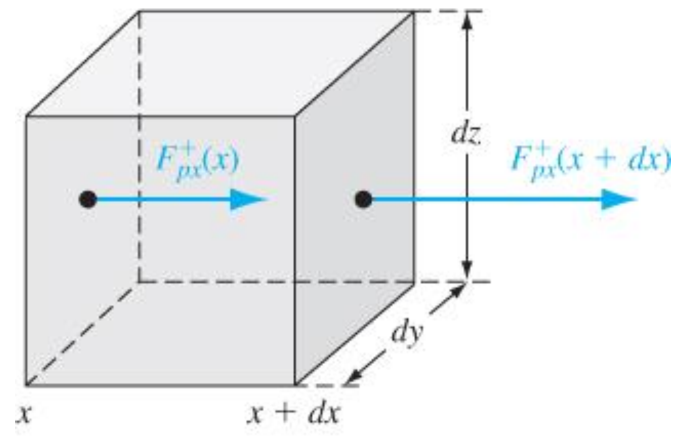


Figure 6.4 | Differential volume showing x component of the hole-particle flux.

The net increase in the number of holes per unit time within the differential volume due to the x-component of hole flux :

$$\frac{\partial p}{\partial t} dx\,dy\,dz = [F_{px}^{+}(x) - F_{px}^{+}(x+dx)]\,dy\,dz = -\frac{\partial F_{px}^{+}}{\partial x} dx\,dy\,dz$$

The generation and recombination rate of holes within the differential volume will also affect the hole concentration :

$$\frac{\partial p}{\partial t} dx\,dy\,dz = -\frac{\partial F_{p}^{+}}{\partial x} dx\,dy\,dz + g_p\,dx\,dy\,dz - \frac{p}{\tau_{pt}} dx\,dy\,dz \Longrightarrow \frac{\partial p}{\partial t} = -\frac{\partial F_{p}^{+}}{\partial x} + g_p - \frac{p}{\tau_{pt}}$$

hole flux + generation - recombination

$$\left(\frac{p}{\tau_{pt}} = \frac{p_0}{\tau_p} + \frac{\delta p}{\tau_{p0}} \right)$$

$$\frac{\partial n}{\partial t} = -\frac{\partial F_{n}^{-}}{\partial x} + g_n - \frac{n}{\tau_{nt}}$$

: for electron

thermal equilibrium carrier lifetime

excess carrier lifetime

Time-Dependent Diffusion Equations

From continuity equations

$$J_p = e\mu_p p\mathrm{E} - eD_p\frac{\partial p}{\partial x} \qquad \frac{J_p}{(+e)} = F_p^+ = \mu_p p\mathrm{E} - D_p\frac{\partial p}{\partial x} \qquad \frac{\partial p}{\partial t} = -\mu_p\frac{\partial (p\mathrm{E})}{\partial x} + D_p\frac{\partial^2 p}{\partial x^2} + g_p - \frac{p}{\tau_{pt}}$$

$$J_n = e\mu_n n\mathrm{E} + eD_n\frac{\partial n}{\partial x} \qquad \frac{J_n}{(-e)} = F_n^- = -\mu_n n\mathrm{E} - D_n\frac{\partial n}{\partial x} \qquad \frac{\partial n}{\partial t} = +\mu_n\frac{\partial (n\mathrm{E})}{\partial x} + D_n\frac{\partial^2 n}{\partial x^2} + g_n - \frac{n}{\tau_{nt}}$$

$$D_p\frac{\partial^2 p}{\partial x^2} - \mu_p\left(\mathrm{E}\frac{\partial p}{\partial x} + p\frac{\partial \mathrm{E}}{\partial x}\right) + g_p - \frac{p}{\tau_{pt}} = \frac{\partial p}{\partial t}$$

$$D_n\frac{\partial^2 n}{\partial x^2} + \mu_n\left(\mathrm{E}\frac{\partial n}{\partial x} + n\frac{\partial \mathrm{E}}{\partial x}\right) + g_n - \frac{n}{\tau_{nt}} = \frac{\partial n}{\partial t}$$

: determine the space and time behavior of the excess carriers !!

$$D_p\frac{\partial^2 (\delta p)}{\partial x^2} - \mu_p\left(\mathrm{E}\frac{\partial (\delta p)}{\partial x} + p\frac{\partial \mathrm{E}}{\partial x}\right) + g_p - \frac{p}{\tau_{pt}} = \frac{\partial (\delta p)}{\partial t}$$

$$D_n\frac{\partial^2 (\delta n)}{\partial x^2} + \mu_n\left(\mathrm{E}\frac{\partial (\delta n)}{\partial x} + n\frac{\partial \mathrm{E}}{\partial x}\right) + g_n - \frac{n}{\tau_{nt}} = \frac{\partial (\delta n)}{\partial t}$$

For homogeneous semiconductor, n_0 and p_0 are independent of the space.

$$n(t) = n_0 + \delta n(t)$$

$$p(t) = p_0 + \delta p(t)$$

$$\mathrm{E} = \mathrm{E}_{\mathrm{app}} + \mathrm{E}_{\mathrm{int}}$$

Since the internal E-field creates a force attracting the electrons and holes, this E-field will hold the pulses of excess electrons and excess holes together. → The electrons and holes will drift or diffuse together with a single effective mobility or diffusion coefficient : Ambipolar transport !

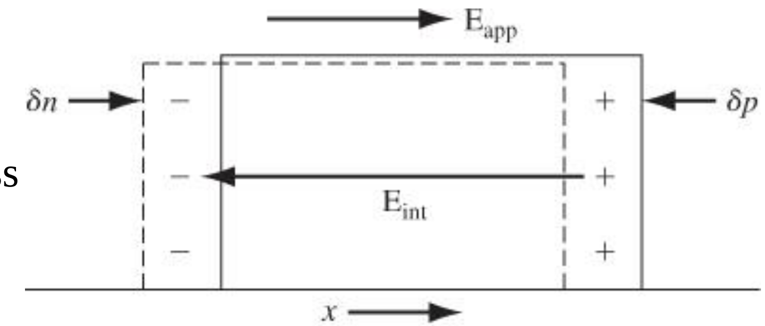


Figure 6.5 | The creation of an internal electric field as excess electrons and holes *tend* to separate.

Derivation of the Ambipolar Transport Equation

$$g_n = g_p \equiv g$$

$$R_n = \frac{n}{\tau_{nt}} = R_p = \frac{p}{\tau_{pt}} \equiv R$$

$$\delta n \approx \delta p$$

$$D_p \frac{\partial^2(\delta n)}{\partial x^2} - \mu_p \left(\mathrm{E} \frac{\partial(\delta n)}{\partial x} + p \frac{\partial \mathrm{E}}{\partial x} \right) + g - R = \frac{\partial(\delta n)}{\partial t} \qquad \times \mu_n n$$

$$\oplus$$

$$D_n \frac{\partial^2(\delta n)}{\partial x^2} + \mu_n \left(\mathrm{E} \frac{\partial(\delta n)}{\partial x} + n \frac{\partial \mathrm{E}}{\partial x} \right) + g - R = \frac{\partial(\delta n)}{\partial t} \qquad \times \mu_p p$$

⇒

$$(\mu_n n D_p + \mu_p p D_n) \frac{\partial^2(\delta n)}{\partial x^2} + (\mu_n \mu_p)(p - n) \mathrm{E} \frac{\partial(\delta n)}{\partial x}$$

$$+ (\mu_n n + \mu_p p)(g - R) = (\mu_n n + \mu_p p) \frac{\partial(\delta n)}{\partial t}$$

$$\div (\mu_n n + \mu_p p)$$

⇒

Ambipolar Transport Equation !

$$D' \frac{\partial^2(\delta n)}{\partial x^2} + \mu' \mathrm{E} \frac{\partial(\delta n)}{\partial x} + g - R = \frac{\partial(\delta n)}{\partial t}$$

$$\mu' = \frac{\mu_n \mu_p (p - n)}{\mu_n n + \mu_p p} \qquad D' = \frac{D_n D_p (n + p)}{D_n n + D_p p}$$

For extrinsic semiconductor and by considering low-level injection :

For a p-type material ($p_0 \gg n_0$) and $\delta n \ll p_0$,

$$D' = \frac{D_n D_p (n+p)}{D_n n + D_p p} = \frac{D_n D_p [(n_0 + \delta n) + (p_0 + \delta n)]}{D_n (n_0 + \delta n) + D_p (p_0 + \delta n)} = D_n$$

Ambipolar diffusion coefficient and mobility → minority carrier parameters (constants)

$$\mu' = \frac{\mu_n \mu_p (p-n)}{\mu_n n + \mu_p p} = \mu_n$$

$$g - R = g_n - R_n = (G_{n0} + g_n') - (R_{n0} + R_n')$$

$$G_{n0} = R_{n0}$$

$$g - R = g_n' - R_n' = g_n' - \frac{\delta n}{\tau_n}$$

⇨ $$D_n \frac{\partial^2 (\delta n)}{\partial x^2} + \mu_n \mathrm{E} \frac{\partial (\delta n)}{\partial x} + g' - \frac{\delta n}{\tau_{n0}} = \frac{\partial (\delta n)}{\partial t}$$

: describe the drift, diffusion, and recombination of excess minority carriers as a function of spatial coordinates and as a function of time.

Similarly,

$$D_p \frac{\partial^2 (\delta p)}{\partial x^2} - \mu_p \mathrm{E} \frac{\partial (\delta p)}{\partial x} + g' - \frac{\delta p}{\tau_{p0}} = \frac{\partial (\delta p)}{\partial t}$$

Example 6.1 : For an infinite and homogeneous n-type semiconductor, a uniform concentration of excess carriers at t =0 and g'=0 for t >0. Low level injection and no E-field.

$$\partial^2(\delta p)/\partial x^2 = \partial(\delta p)/\partial x = 0$$

The ambipolar transport equation for the minority carrier holes,

$$D_p \frac{\partial^2(\delta p)}{\partial x^2} - \mu_p \mathrm{E} \frac{\partial(\delta p)}{\partial x} + g' - \frac{\delta p}{\tau_{p0}} = \frac{\partial(\delta p)}{\partial t} \Longrightarrow \frac{d(\delta p)}{dt} = -\frac{\delta p}{\tau_{p0}} \Longrightarrow \delta p(t) = \delta p(0) e^{-t/\tau_{p0}}$$

Example 6.2 : For an infinite and homogeneous n-type semiconductor, a uniform generation rate at $t \geq 0$. Low level injection and no E-field.

$$g' - \frac{\delta p}{\tau_{p0}} = \frac{d(\delta p)}{dt} \Longrightarrow \delta p(t) = g' \tau_{p0} (1 - e^{-t/\tau_{p0}})$$

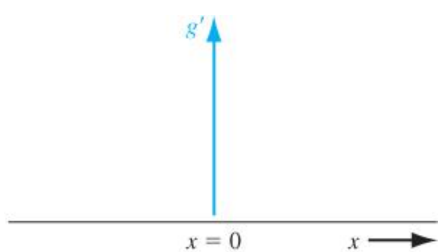
$$\delta p(t) = (5 \times 10^{21})(10^{-7})[1 - e^{-t/10^{-7}}] = 5 \times 10^{14}\,[1 - e^{-t/10^{-7}}]\ \mathrm{cm}^{-3}$$

Example 6.3 : For an infinite and homogeneous p-type semiconductor, the excess carriers are generated only at x =0 position. Low level injection and no E-field. Steady-state excess carrier concentration ?

$$D_n \frac{\partial^2(\delta n)}{\partial x^2} + \mu_n \mathrm{E} \frac{\partial(\delta n)}{\partial x} + g' - \frac{\delta n}{\tau_{n0}} = \frac{\partial(\delta n)}{\partial t} \Longrightarrow D_n \frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{\tau_{n0}} = 0$$

$$\Longrightarrow \frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{D_n \tau_{n0}} = \frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{L_n^2} = 0 \Longrightarrow \delta n(x) = A e^{-x/L_n} + B e^{x/L_n}$$

$$\Longrightarrow \begin{cases} \delta n(x) = \delta n(0) e^{+x/L_n} & x \leq 0 \\ \delta n(x) = \delta n(0) e^{-x/L_n} & x \geq 0 \end{cases}$$



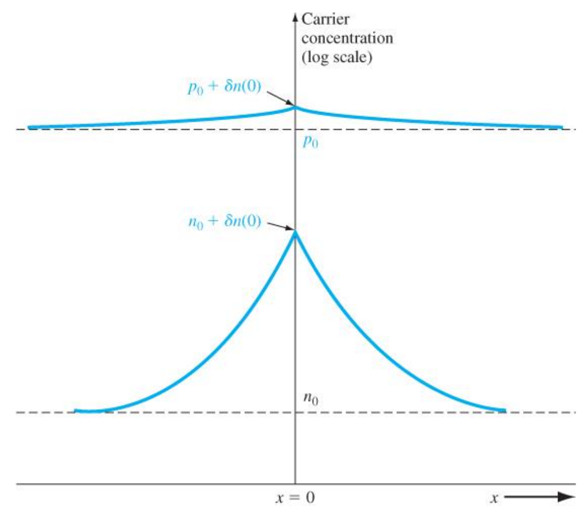


Figure 6.7 | Steady-state electron and hole concentrations for the case when excess electrons and holes are generated at $x = 0$.

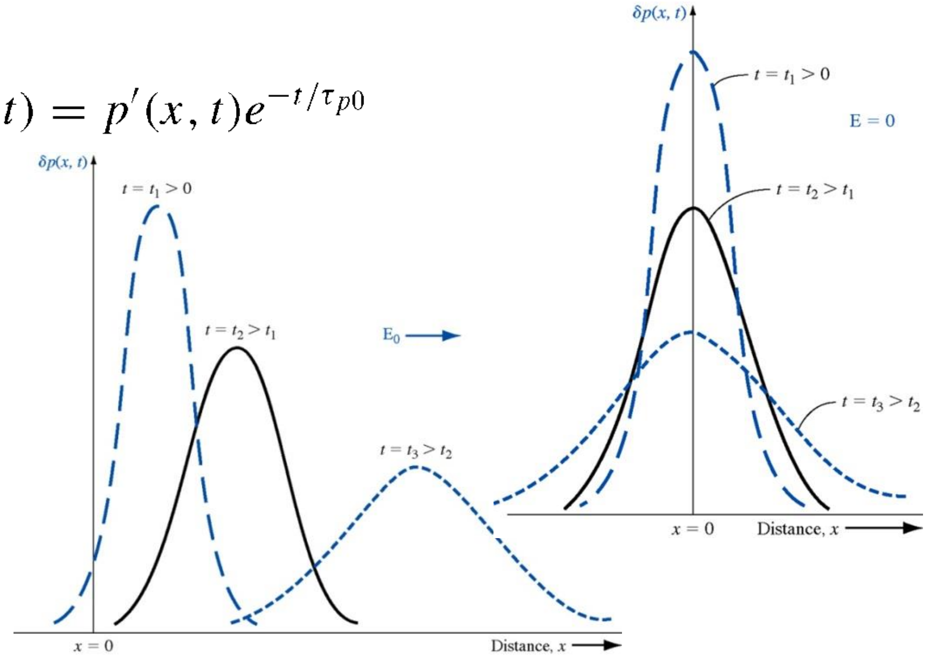
Example 6.4 : For an infinite and homogeneous n-type semiconductor, a finite number of electron-hole pairs is generated instantaneously at time t =0 and at x =0, but g'=0 for t >0. Low level injection and constant E-field E_0 in the +x direction.

$$D_p \frac{\partial^2(\delta p)}{\partial x^2} - \mu_p E_0 \frac{\partial(\delta p)}{\partial x} - \frac{\delta p}{\tau_{p0}} = \frac{\partial(\delta p)}{\partial t} \Rightarrow \delta p(x,t) = p'(x,t)e^{-t/\tau_{p0}}$$

$$\Rightarrow D_p \frac{\partial^2 p'(x,t)}{\partial x^2} - \mu_p E_0 \frac{\partial p'(x,t)}{\partial x} = \frac{\partial p'(x,t)}{\partial t}$$

$$\Rightarrow p'(x,t) = \frac{1}{(4\pi D_p t)^{1/2}} \exp\left[\frac{-(x-\mu_p E_0 t)^2}{4D_p t}\right]$$

$$\Rightarrow \delta p(x,t) = \frac{e^{-t/\tau_{p0}}}{(4\pi D_p t)^{1/2}} \exp\left[\frac{-(x-\mu_p E_0 t)^2}{4D_p t}\right]$$



Dielectric Relaxation Time Constant

δp is suddenly injected into a portion of semiconductor → How is charge neutrality achieved and how fast ?

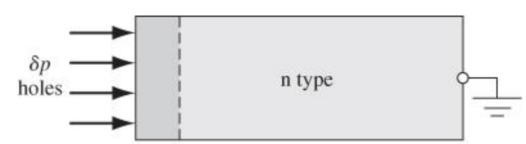


Figure 6.10 | The injection of a concentration of holes into a small region at the surface of an n-type semiconductor.

$$\nabla \cdot E = \frac{\rho}{\epsilon}$$

$$J = \sigma E$$

$$\nabla \cdot J = -\frac{\partial \rho}{\partial t}$$

$$\nabla \cdot J = \sigma \nabla \cdot E = \frac{\sigma \rho}{\epsilon}$$

$$\Rightarrow \frac{\sigma\rho}{\epsilon} = -\frac{\partial \rho}{\partial t} = -\frac{d\rho}{dt}$$

$$\frac{d\rho}{dt} + \left(\frac{\sigma}{\epsilon}\right)\rho = 0$$

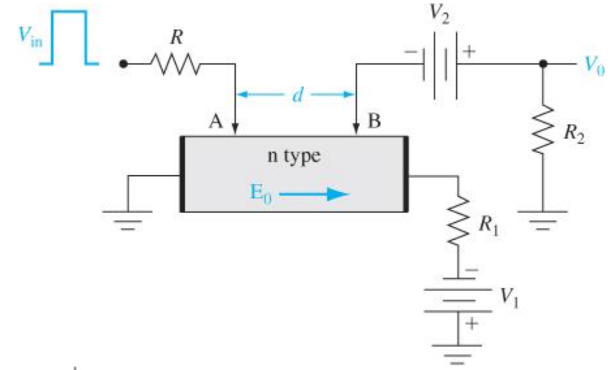
$$\rho(t) = \rho(0)e^{-(t/\tau_d)}$$

$$\tau_d = \frac{\epsilon}{\sigma}$$

: dielectric Relaxation Time Constant

Haynes-Shockley Experiment

Experimental measurement of **mobility**, **diffusion coefficient**, and **minority carrier lifetime**.

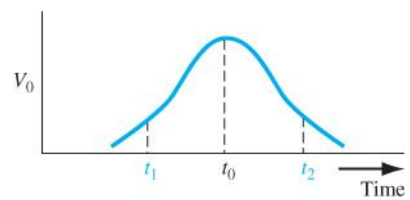


$$\delta p(x,t) = \frac{e^{-t/\tau_{p0}}}{(4\pi D_p t)^{1/2}} \exp\left[\frac{-(x-\mu_p E_0 t)^2}{4D_p t}\right]$$ from Example 6.4

$$x - \mu_p E_0 t = 0 \quad \Longrightarrow \quad \mu_p = \frac{d}{E_0 t_0}$$

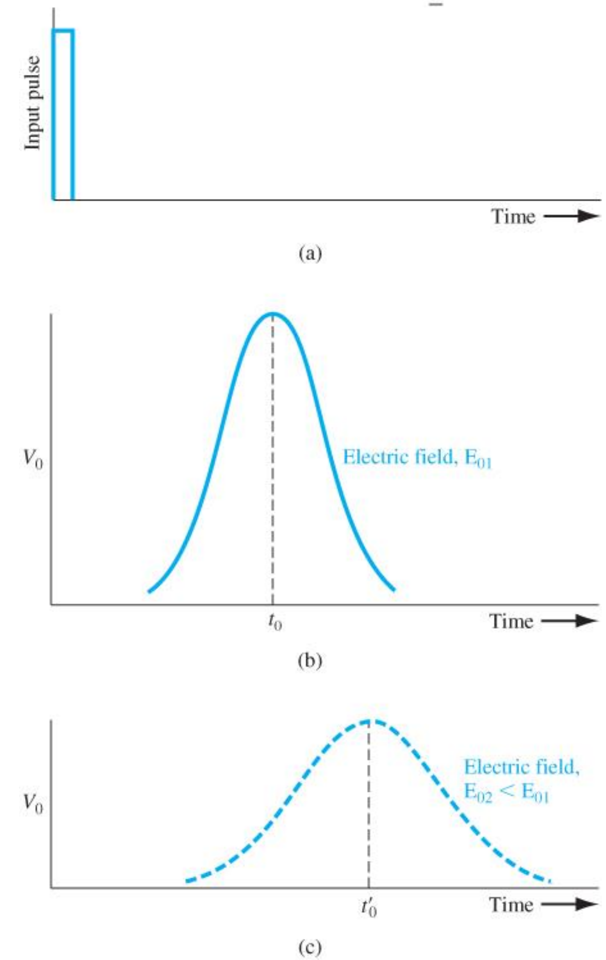
Output V_0 vs. time

t_1, t_2 : e^{-1} of peak and not too long



$$(d - \mu_p E_0 t)^2 = 4D_p t \qquad at\ t = t_1, t_2$$

$$\Longrightarrow \quad D_p = \frac{(\mu_p E_0)^2 (\Delta t)^2}{16 t_0} \qquad \Delta t = t_2 - t_1$$



The area S is proportional to the number of excess holes that not recombined with majority electrons :

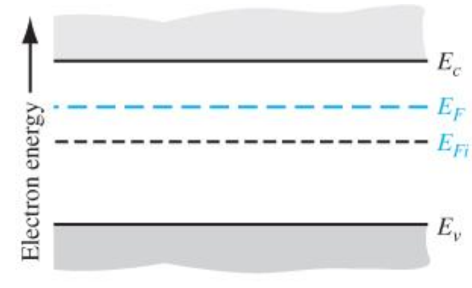
$$S = K \exp\left(\frac{-t_0}{\tau_{p0}}\right) = K \exp\left(\frac{-d}{\mu_p E_0 \tau_{p0}}\right)$$

τ_{p0} can be obtained from the plot of ln$S$ versus ($d/\mu_p E_0$).

Quasi-Fermi Energy Levels

If excess carriers are created in a semiconductor, we are no longer in thermal equilibrium and the Fermi energy level is strictly no longer defined.

→ We may define a quasi-Fermi levels for electron and holes that apply for nonequilibrium.



(a)

(b)

At Equilibrium

$$n_0 = n_i \exp\left(\frac{E_F - E_{Fi}}{kT}\right)$$

$$p_0 = n_i \exp\left(\frac{E_{Fi} - E_F}{kT}\right)$$

At Nonequilibrium

$$n_0 + \delta n = n_i \exp\left(\frac{E_{Fn} - E_{Fi}}{kT}\right)$$

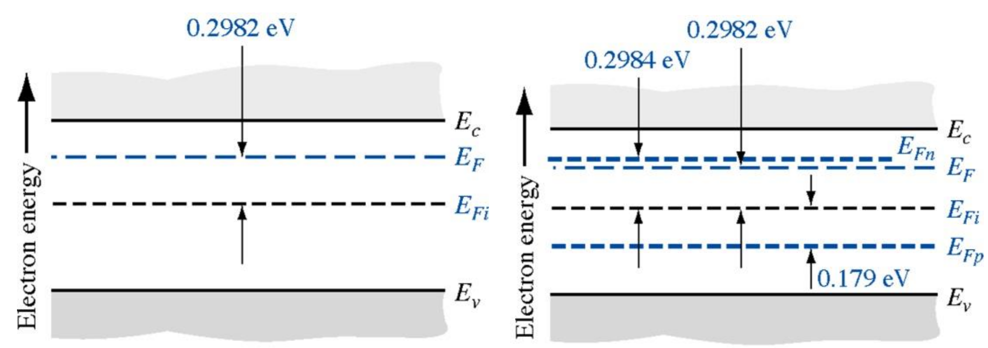
$$p_0 + \delta p = n_i \exp\left(\frac{E_{Fi} - E_{Fp}}{kT}\right)$$

Example 6.6 : An n-type semiconductor at T =300K with n_0 =10^{15}cm^{-3}, n_i =10^{10}cm^{-3}, and p_0 =10^5cm^{-3}. In nonequilibrium, assume that the excess carrier concentrations are $\delta n = \delta p$ =10^{13}cm^{-3}.

$$E_F - E_{Fi} = kT \ln\left(\frac{n_0}{n_i}\right) = 0.2982 \text{ eV}$$

$$E_{Fn} - E_{Fi} = kT \ln\left(\frac{n_0 + \delta n}{n_i}\right) = 0.2984 \text{ eV}$$

$$E_{Fi} - E_{Fp} = kT \ln\left(\frac{p_0 + \delta p}{n_i}\right) = 0.179 \text{ eV}$$



(a) (b)

Shockley-Read-Hall Theory of Recombination

Trap : An allowed energy state within the forbidden bandgap caused by defects or imperfection of periodic potential.

→Acts as a *recombination center* , capturing both electrons and holes with almost equal probability.

→ four basic processes: Electron Capture, Electron Emission, Hole Capture, and Hall Emission

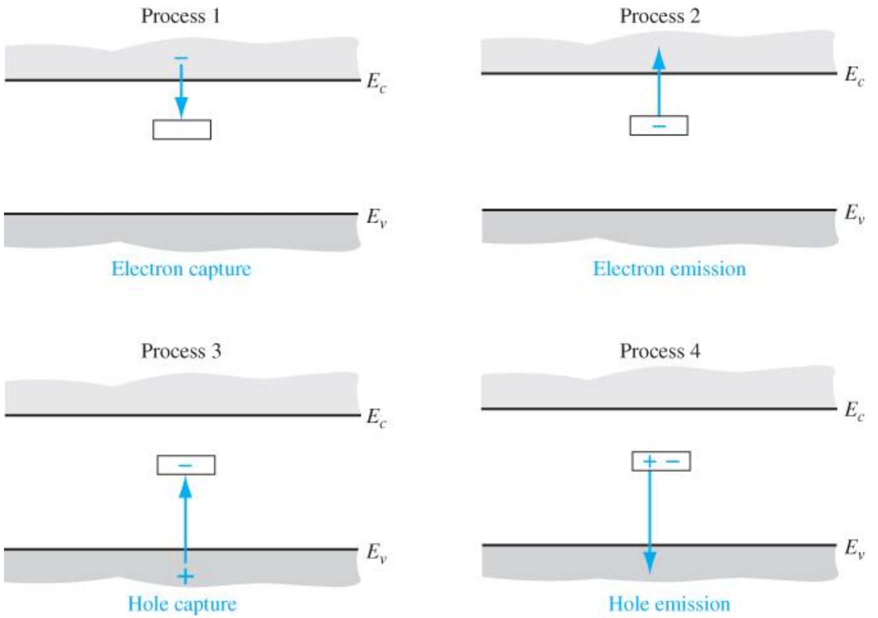


Figure 6.16 | The four basic trapping and emission processes for the case of an acceptor-type trap.

$$R_{cn} = C_n N_t (1 - f_F(E_t))n$$

R_{cn} = capture rate (#/cm^3-s)

C_n = constant proportional to electron-capture cross section

N_t = total concentration of trapping centers

n = electron concentration

$f_F(E_t)$ = Fermi function at the trap energy

$$f_F(E_t) = \frac{1}{1 + \exp\left[\frac{E_t - E_F}{kT}\right]}$$

$$R_{en} = E_n N_t f_F(E_t)$$

R_{en} = emission rate (#/cm^3-s)

E_n = constant proportional to emission cross section

$f_F(E_t)$ = probability that the trap is occupied by electron

In thermal equilibrium, $R_{en} = R_{cn}$

$$\Rightarrow \quad E_n N_t f_{F0}(E_t) = C_n N_t (1 - f_{F0}(E_t)) n_0$$

$f_{F0}(E_t)$ = Thermal equilibrium Fermi function

n_0 = electron concentration at equilibrium

$$\frac{E_n}{C_n} = n' = \frac{n_0\left(1 - f_{F0}(E_t)\right)}{f_{F0}(E_t)} = N_C \exp\left(\frac{E_F - E_C}{kT}\right)\exp\left(\frac{E_t - E_F}{kT}\right) = N_C \exp\left(\frac{-(E_C - E_t)}{kT}\right)$$

In nonequilibrium, the net rate at which electrons are captured from the conduction band is given by :

$$R_n = R_{cn} - R_{en} = [C_n N_t (1 - f_F(E_t))n] - [E_n N_t f_F(E_t)]$$
$$= C_n N_t [n(1 - f_F(E_t)) - n' f_F(E_t)]$$

n includes the excess electron concentration.

$E_F \rightarrow E_{Fn}$

Similarly, the net rate at which holes are captured from the valence band is given by :

$$R_p = C_p N_t [p f_F(E_t) - p'(1 - f_F(E_t))]$$

where $p' = N_v \exp\left[\frac{-(E_t - E_v)}{kT}\right]$

In a semiconductor in which the trap density is not too large, the excess electron and hole concentrations are equal and the recombination rates are equal :

$$R_n = R_p \Rightarrow f_F(E_t) = \frac{C_n n + C_p p'}{C_n(n + n') + C_p(p + p')}$$

$$n' p' = n_i^2$$

Recombination rate due to the trap at $E = E_t$ in case that excess carriers exist

$$R_n = R_p = \frac{C_n C_p N_t \left(np - n_i^2\right)}{C_n(n + n') + C_p(p + p')} \equiv R = \frac{\delta n}{\tau}$$

For an n-type semiconductor under low injection, $n_0 \gg p_0$, $n_0 \gg \delta p$, $n_0 \gg n'$, $n_0 \gg p'$:

$$R \cong \frac{C_n C_p N_t (\delta p \cdot n_0 + \delta n \cdot p_0 + \delta p \cdot \delta n)}{C_n \cdot n_0} = C_p N_t \delta p \equiv \frac{\delta p}{\tau_{p0}}$$

where $\tau_{p0} = \frac{1}{C_p N_t}$

: function of minority carrier parameters