Theory of Semiconductor Devices
(반도체 소자 이론)

Lecture 16

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p-n Junction

Introduction

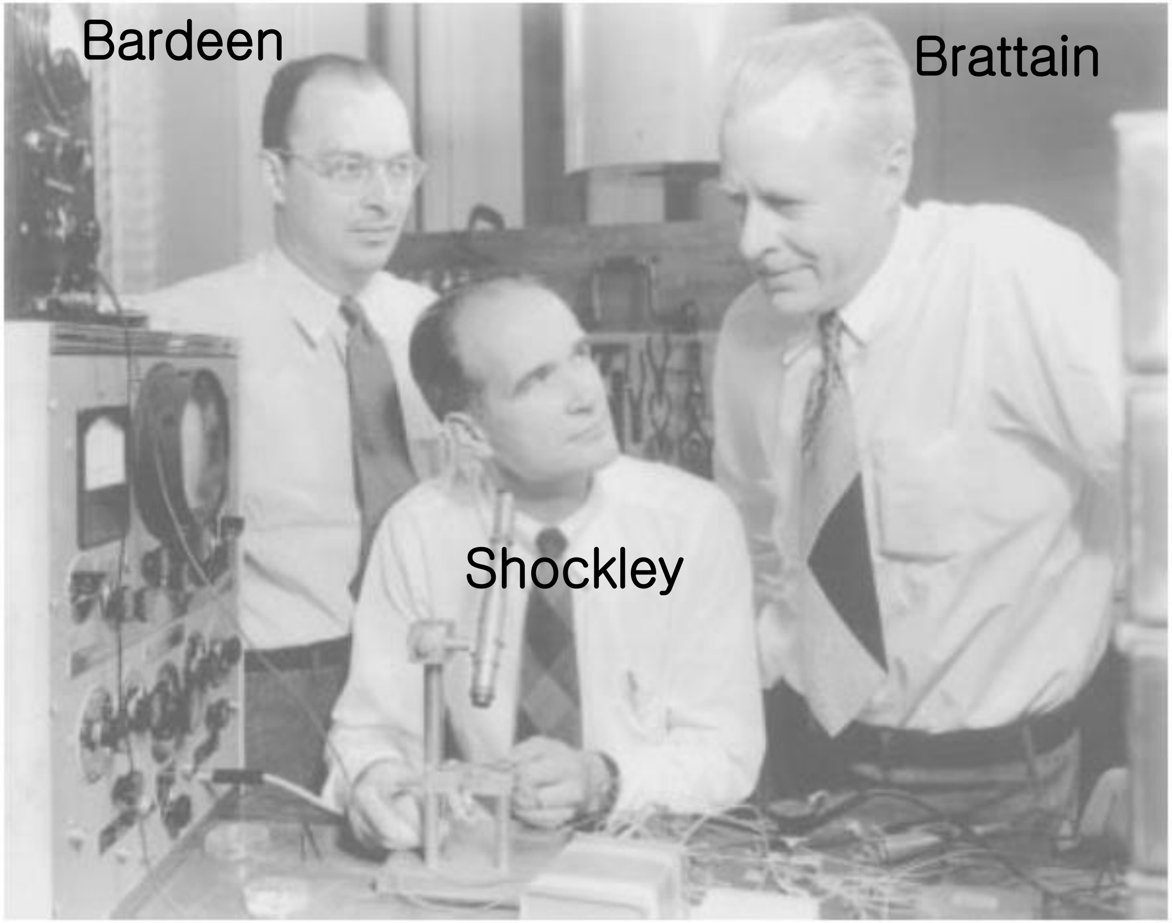
- *p-n* junctions are of great importance both in modern electronic applications and in understanding other semiconductor devices.
- The p-n junction theory serves as the ***foundation*** of the physics of semiconductor devices.
-The basic theory of current-voltage characteristics of *p-n* junctions was established by **Shockley**. This theory was then extended by Sah, Noyce, and Shockley, and by Moll.

- The *basic* equations (previous class) are used to develop the ***ideal*** static and dynamic characteristics of *p-n* junctions.
- Departures *from* the ideal characteristics due to generation and recombination in the ***depletion*** layer, *to* high injection, and *to* series resistance effects are then discussed. Junction breakdown, especially that due to avalanche multiplication, is considered in detail, after which transient behavior and noise performance in *p-n* junctions are presented.

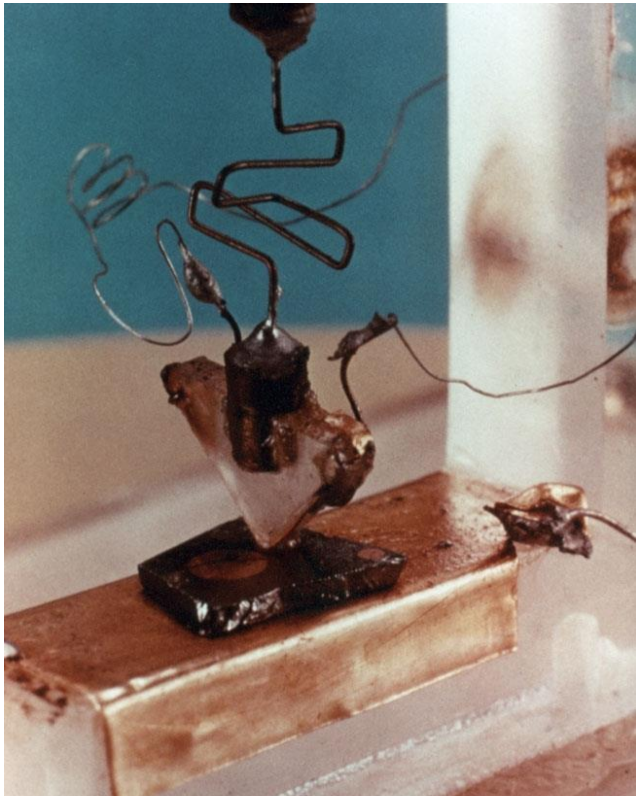
- A *p-n* junction is a ***two-terminal*** device. Depending on the doping profile, device geometry, and biasing condition, a p-n junction can perform *various* terminal functions which are considered briefly in Section 2.6.
- The chapter closes with a discussion of an important group of devices-the ***heterojunctions***, which are junctions formed between dissimilar semiconductors (e.g., *n*-type GaAs on *p*-type AlGaAs).

Introduction

Bardeen, Shockley, and Brattain



Nobel Prize (Physics) @1956



The first point-contact Transistor invented.

Bell Telephone Lab.
Dec. 23, 1947

Contents

p-n Junction

Depletion Region: Abrupt Junction

Built-in Potential and Depletion-Layer Width

- When the impurity concentration in a semiconductor **changes abruptly** from acceptor impurities N_A to donor impurities N_D, as shown in Fig. l(a), one obtains an abrupt junction.
- In particular, if $N_A \gg N_D$ (or vice versa), one obtains a one-sided abrupt p^+-n (or n^+-p) junction.

- We first consider the **thermal equilibrium** condition, that is, one ***without*** applied voltage and current flow. From the current equation of drift and diffusion

$$J_n = 0 = q\mu_n\left(n\mathscr{E} + \frac{kT}{q}\frac{dn}{dx}\right) = \mu_n n\frac{dE_F}{dx} \qquad (1)$$

or

$$\frac{dE_F}{dx} = 0\,. \qquad (2)$$

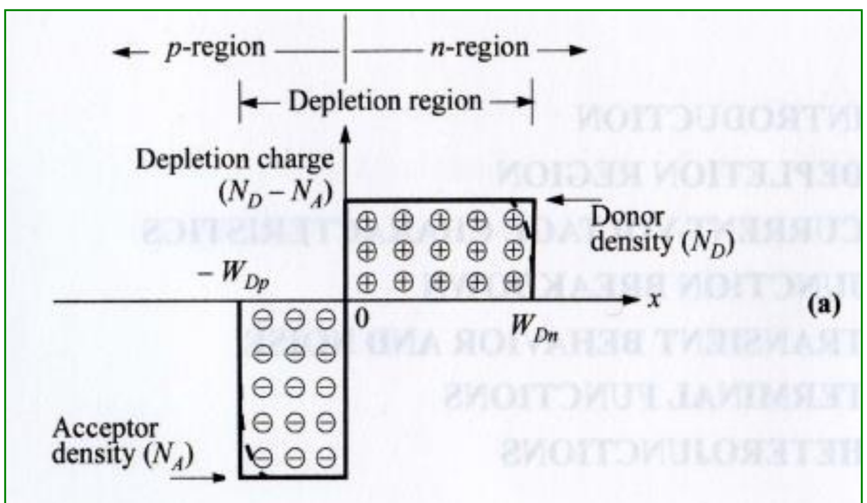


Fig. 1 Abrupt *p-n* junction in thermal equilibrium. (a) Space-charge distribution. Dashed lines indicate corrections to depletion approximation.

Abrupt Junction

- Similarly,

$$J_p = 0 = \mu_p p \frac{dE_F}{dx} . \qquad (3)$$

- Thus the condition of **zero** net electron and hole currents requires that the Fermi level ***must be*** constant throughout the sample. The **built-in potential** ψ_{bi} , or diffusion potential, as shown in Fig. 1 b, c, and d, is equal to

$$q\psi_{bi} = E_g - (q\phi_n + q\phi_p) = q\psi_{Bn} + q\psi_{Bp} . \qquad (4)$$

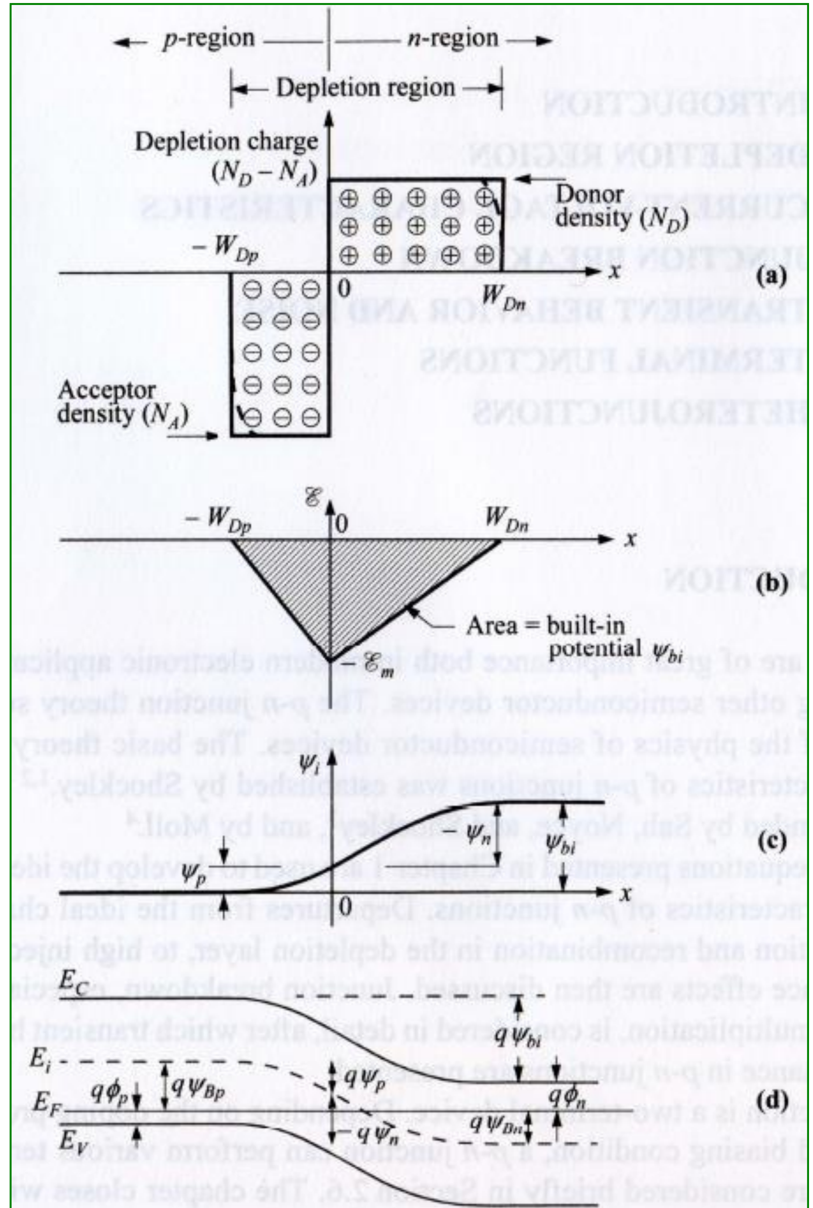


Fig. 1 Abrupt *p-n* junction in thermal equilibrium.
(a) Space-charge distribution. Dashed lines indicate corrections to depletion approximation.
(b) Electric-field distribution.
(c) Potential distribution where $f\psi_{bi}$ is the built-in potential. (d) Energy-band diagram.

Abrupt Junction

- For nondegenerate semiconductors,

$$\psi_{bi} = \frac{kT}{q}\ln\left(\frac{n_{no}}{n_i}\right) + \frac{kT}{q}\ln\left(\frac{p_{po}}{n_i}\right) \approx \frac{kT}{q}\ln\left(\frac{N_D N_A}{n_i^2}\right) . \quad (5)$$

- Since at **equilibrium** $n_{no}\,p_{no} = n_{po}\,p_{po} = n_i^2$,

$$\psi_{bi} = \frac{kT}{q}\ln\left(\frac{p_{po}}{p_{no}}\right) = \frac{kT}{q}\ln\left(\frac{n_{no}}{n_{po}}\right) . \quad (6)$$

This gives the relationship between carrier densities on either side of the junction.

- If one or both sides of the junction are **degenerate**, ***care*** has to be taken in calculating the Fermi-levels and built-in potential.
- Equation (4) has to be used since Boltzmann statistics *cannot* be used to simplify the Fermi-Dirac integral.

- Furthermore, ***incomplete*** ionization has to be considered, i.e., $n_{no} \neq N_D$ and/or $p_{po} \neq N_A$.

Abrupt Junction

- Next, we proceed to calculate the field and potential distribution inside the **depletion region**.
- To *simplify* the analysis, the depletion approximation is used which assumes that the depleted charge has a box profile.
- Since in **thermal equilibrium** the ***electric field*** in the neutral regions (*far from* the junction at either side) of the Semiconductor must be ***zero***, the total negative charge per unit area in the p-side must be precisely equal to the total positive charge per unit area in the *n*-side:

$$N_A W_{Dp} = N_D W_{Dn} \,. \quad (7)$$

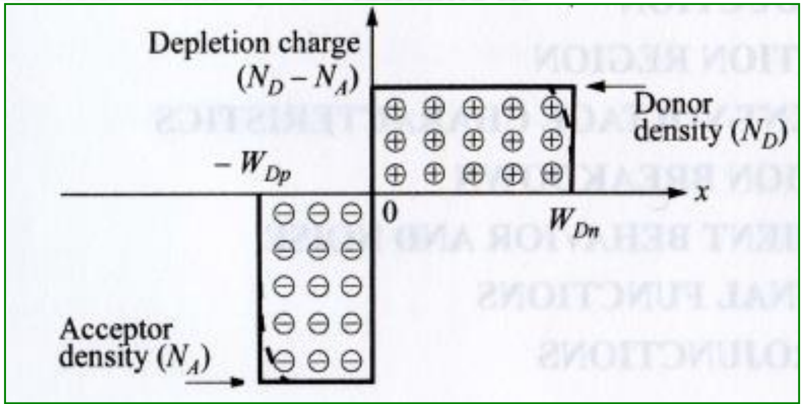
- From the Poisson equation we obtain

$$-\frac{d^2 \psi_i}{dx^2} = \frac{d\mathscr{E}}{dx} = \frac{\rho(x)}{\varepsilon_s} = \frac{q}{\varepsilon_s}[N_D^+(x) - n(x) - N_A^-(x) + p(x)] \,. \quad (8)$$

- Inside the depletion region, $n(x) \approx p(x) \approx 0$,
and ***assuming*** complete ionization,

$$\frac{d^2 \psi_i}{dx^2} \approx \frac{qN_A}{\varepsilon_s} \quad \text{for} \quad -W_{Dp} \le x \le 0 \,, \quad (9a)$$

$$-\frac{d^2 \psi_i}{dx^2} \approx \frac{qN_D}{\varepsilon_s} \quad \text{for} \quad 0 \le x \le W_{Dn} \,. \quad (9b)$$



Abrupt Junction

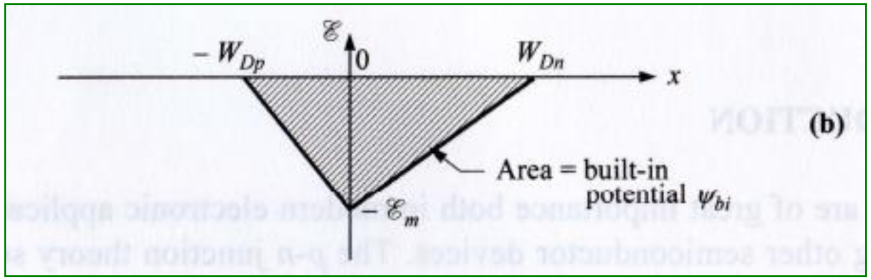
- The electric field is then obtained by integrating the above equations, as shown in Fig. 1b:

$$\mathscr{E}(x) = -\frac{qN_A(x+W_{Dp})}{\varepsilon_s} \quad \text{for } -W_{Dp} \le x \le 0, \tag{10}$$

$$\mathscr{E}(x) = -\mathscr{E}_m + \frac{qN_D x}{\varepsilon_s} = -\frac{qN_D}{\varepsilon_s}(W_{Dn}-x) \quad \text{for } 0 \le x \le W_{Dn} \tag{11}$$

where $\mathscr{E}_m$ is the maximum field that exists at $x = 0$ and is given by

$$|\mathscr{E}_m| = \frac{qN_D W_{Dn}}{\varepsilon_s} = \frac{qN_A W_{Dp}}{\varepsilon_s}. \tag{12}$$



-Integrating Eqs. 10 and 11 once again gives the potential distribution $\psi_i(x)$ (Fig. 1c)

$$\psi_i(x) = \frac{qN_A}{2\varepsilon_s}(x+W_{Dp})^2 \quad \text{for } -W_{Dp} \le x \le 0, \tag{13}$$

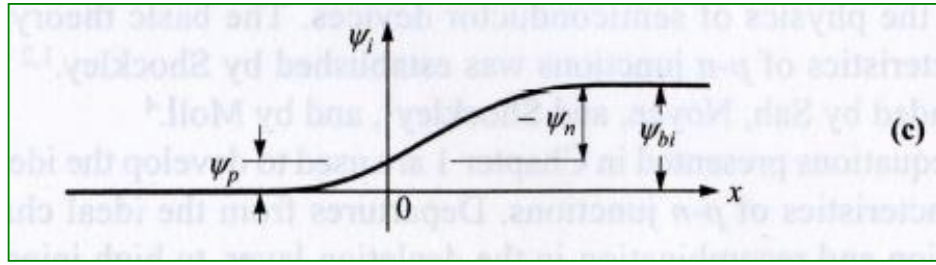
$$\psi_i(x) = \psi_i(0) + \frac{qN_D}{\varepsilon_s}\left(W_{Dn} - \frac{x}{2}\right)x \quad \text{for } 0 \le x \le W_{Dn}. \tag{14}$$

Abrupt Junction

- With these, the potentials across different regions can be found as:

$$\psi_p = \frac{qN_A W_{Dp}^2}{2\varepsilon_s}, \qquad (15a)$$

$$|\psi_n| = \frac{qN_D W_{Dn}^2}{2\varepsilon_s}, \qquad (15b)$$



(ψ_n is relative to the *n*-type bulk and is thus negative.)

$$\psi_{bi} = \psi_p + |\psi_n| = \psi_i(W_{Dn}) = \frac{|\mathscr{E}_m|}{2}(W_{Dp} + W_{Dn}) \qquad (16)$$

where $\mathscr{E}_m$ can also be expressed as:

$$|\mathscr{E}_m| = \sqrt{\frac{2qN_A\psi_p}{\varepsilon_s}} = \sqrt{\frac{2qN_D|\psi_n|}{\varepsilon_s}}. \qquad (17)$$

From Eqs. 16 and 7, the depletion widths are calculated to be:

$$W_{Dp} = \sqrt{\frac{2\varepsilon_s\psi_{bi}}{q}\frac{N_D}{N_A(N_A+N_D)}} \qquad (18a)$$

$$W_{Dn} = \sqrt{\frac{2\varepsilon_s\psi_{bi}}{q}\frac{N_A}{N_D(N_A+N_D)}}, \qquad (18b)$$

$$W_{Dp} + W_{Dn} = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{N_A+N_D}{N_AN_D}\right)\psi_{bi}}. \qquad (19)$$

Abrupt Junction

- The following relationships can be further deduced:

$$\frac{|\psi_n|}{\psi_{bi}} = \frac{W_{Dn}}{W_{Dp} + W_{Dn}} = \frac{N_A}{N_A + N_D}, \qquad (20a)$$

$$\frac{\psi_p}{\psi_{bi}} = \frac{W_{Dp}}{W_{Dp} + W_{Dn}} = \frac{N_D}{N_A + N_D}. \qquad (20b)$$

- For a *one-sided* abrupt junction (p^+-n or n^+-p), Eq. 4 is used to calculate the built-in potential.
- In this case, the ***majority*** of the potential variation and depletion region will be inside the ***lightly doped*** side.
- Equation 19 reduces to

$$W_D = \sqrt{\frac{2\varepsilon_s \psi_{bi}}{qN}} \qquad (21)$$

where N is N_D or N_A depending on whether $N_A \gg N_D$ or vice versa, and

$$\psi_i(x) = |\mathscr{E}_m|\left(x - \frac{x^2}{2W_D}\right). \qquad (22)$$

This discussion uses box profiles for the depletion charges, i.e., depletion approximation.

Abrupt Junction

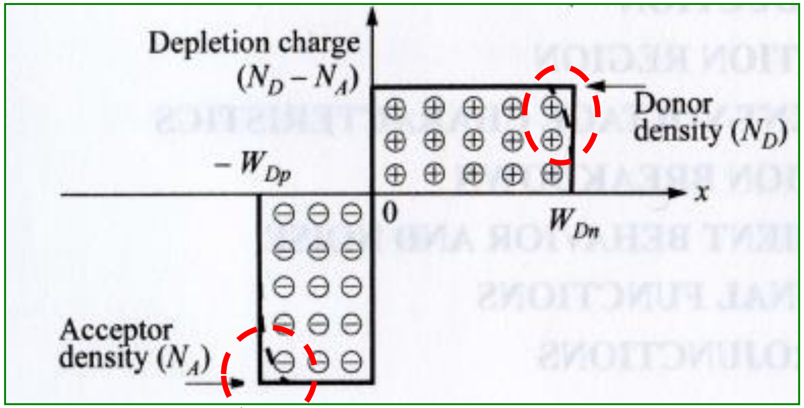
- A ***more accurate*** result for the depletion-layer properties can be obtained by considering the ***majority***-carrier contribution in addition to the impurity concentration in the Poisson equation, that is, $\rho \approx -q[N_A - p(x)]$ on the *p*-side and $\rho \approx q[N_D - n(x)]$ on the *n*-side.

- The depletion width is essentially the ***same*** as given by Eq. 19, ***except*** that ψ_{bi} is replaced by $(\psi_{bi} - 2kT/q)$.

- The correction factor *2kT/q* comes about because of the two majority-carrier distribution tails (electrons in *n*-side and holes in *p*-side, as shown by the dashed lines in Fig. l (a) near the edges of the depletion region. Each contributes a correction factor *kT/q*.

- The depletion-layer width at thermal equilibrium for a one-sided abrupt junction becomes

$$W_D = \sqrt{\frac{2\varepsilon_s}{qN}\left(\psi_{bi} - \frac{2kT}{q}\right)}. \quad (23)$$



Abrupt Junction

With applied bias

- Furthermore, when a voltage V is applied to the junction, the total electrostatic potential variation across the junction is given by $(\psi_{bi} - V)$ where V is positive for forward bias (positive voltage on p-region with respect to n-region) and negative for reverse bias.

- Substituting $(\psi_{bi} - V)$ for ψ_{bi} in Eq. 23 yields the depletion-layer width as a function of the applied voltage. The results for one-sided abrupt junctions in silicon are shown in Fig. 2.

- The net potential at zero bias is near 0.8 V for Si and 1.3 V for GaAs. This net potential will be ***decreased*** under ***forward*** bias and ***increased*** under ***reverse*** bias.

-These results can also be used for GaAs since both Si and GaAs have approximately the same static dielectric constants.

- To obtain the depletion-layer width for other semiconductors such as Ge, one must multiply the results of Si by the factor $\sqrt{\varepsilon_s(\text{Ge})/\varepsilon_s(\text{Si})}$ (= 1.16).

- The simple model above can give adequate predictions for most abrupt *p-n* junctions.

Abrupt Junction

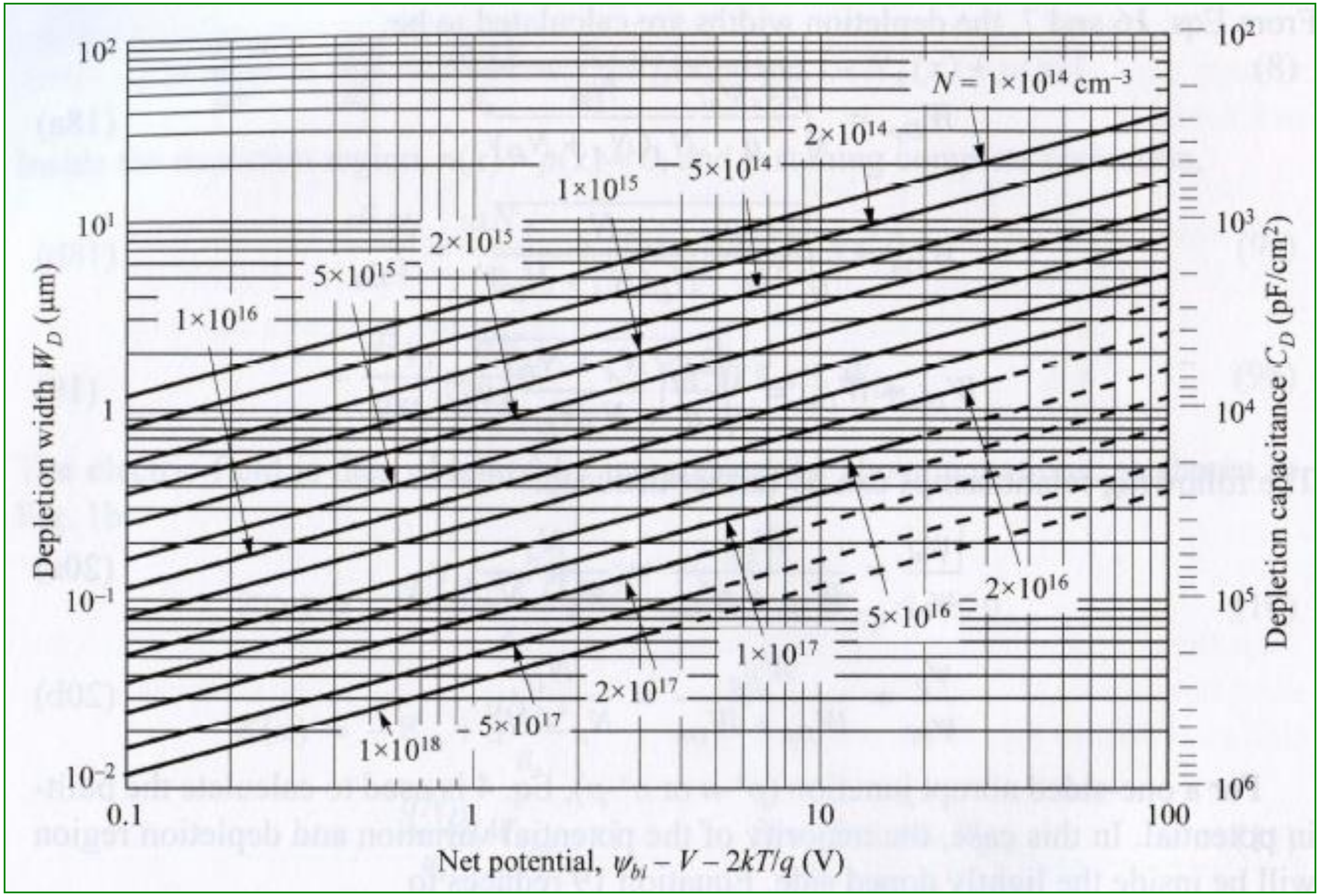


Fig.2 Depletion-layer width and depletion-layer capacitance per unit area as a function of net potential ($\psi_{bi} - V - 2kT/q$) for one-sided abrupt junctions in Si. Doping N is from the lightly doped side. Dashed lines represent breakdown conditions.

Depletion Layer Capacitance

- The depletion-layer capacitance per unit area is defined as $C_D = dQ_D/dV = \varepsilon_s/W_D$, where dQ_D is the *incremental* depletion charge on each side of the junction (total charge is zero) upon an incremental change of the applied voltage dV.
- For one-sided abrupt junctions, the capacitance per unit area is given by

$$C_D = \frac{\varepsilon_s}{W_D} = \sqrt{\frac{q\varepsilon_s N}{2}}\left(\psi_{bi} - V - \frac{2kT}{q}\right)^{-1/2} \quad (24)$$

where V is positive/negative for forward/reverse bias. The results of the depletion layer capacitance are also shown in Fig. 2.
- Rearrange the above equation leads to:

$$\frac{1}{C_D^2} = \frac{2}{q\varepsilon_s N}\left(\psi_{bi} - V - \frac{2kT}{q}\right), \quad (25)$$

$$\frac{d(1/C_D^2)}{dV} = -\frac{2}{q\varepsilon_s N}. \quad (26)$$

- It is clear from Eqs. 25 and 26 that by plotting $1/C^2$ versus V, a straight line should result from a one-sided abrupt junction (Fig. 3). The slope gives the impurity concentration of the substrate (N), and the extrapolation to $1/C^2 = 0$ gives (ψ_{bi} - $2kT/q$). Note that, for the forward bias, a diffusion capacitance exists in addition to the depletion capacitance mentioned previously. The diffusion capacitance will be discussed in Section 2.3.4.

Depletion Layer Capacitance

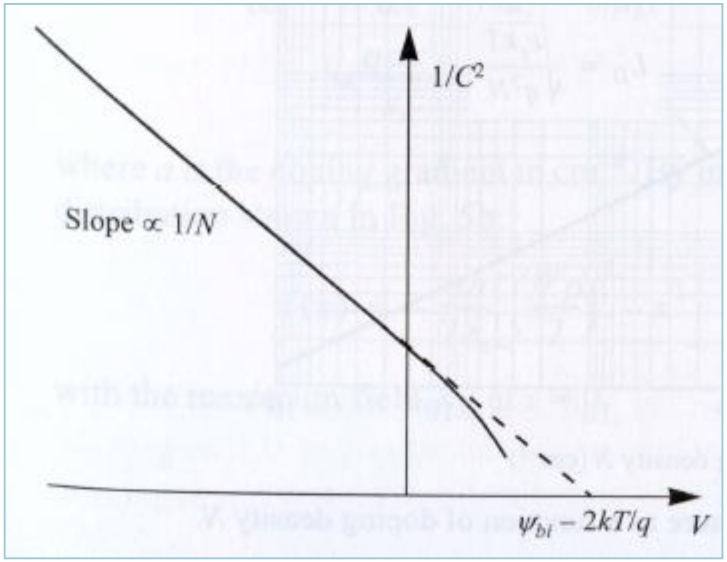


Fig. 3 A $1/C^2 - V$ plot can yield the built-in potential and doping density N.

Note that the semiconductor potential and the capacitance-voltage data are insensitive to changes in the doping profiles that occur in a distance less than a Debye length. The Debye length L_D is a characteristic length for semiconductors and is defined as

$$L_D \equiv \sqrt{\frac{\varepsilon_s kT}{q^2 N}}. \qquad (27)$$

- This Debye length gives an idea of the limit of the potential change in response to an abrupt change in the doping profile.
- Consider a case where the doping has a small increase of ΔN_D in the background of N_D, the change of potential $\Delta\psi_i$ (x) near the step is given by

$$n = N_D \exp\left(\frac{\Delta\psi_i q}{kT}\right), \qquad (28)$$

Depletion Layer Capacitance

$$\frac{d^2\Delta\psi_i}{dx^2} = -\frac{q}{\varepsilon_s}(N_D+\Delta N_D-n) = -\frac{qN_D}{\varepsilon_s}\left[1+\frac{\Delta N_D}{N_D}-\exp\left(\frac{\Delta\psi_i q}{kT}\right)\right]$$

$$\approx -\frac{qN_D}{\varepsilon_s}\left[1+\frac{\Delta N_D}{N_D}-\left(1+\frac{\Delta\psi_i q}{kT}\right)\right] \approx \frac{q^2N_D}{\varepsilon_s kT}\Delta\psi_i \qquad (29)$$

whose solution has a decay length given by Eq. 27.

-This implies that if the doping profile changes ***abruptly*** in a scale less than the Debye length, this variation has ***no effect*** and ***cannot be resolved***, and that if the depletion width is smaller than the Debye length, the analysis using the ***Poisson equation*** is ***no longer valid***.

- At **thermal equilibrium** the depletion-layer widths of abrupt junctions are about $8L_D$ for Si, and $10L_D$ for GaAs. The Debye length as a function of doping density is shown in Fig. 4 for silicon at room temperature.
- For a doping density of 10^{16} cm^{-3}, the Debye length is 40 nm; for other dopings, L_D will vary as $1/\sqrt{N}$, that is, a reduction by a factor of 3.16 per decade.

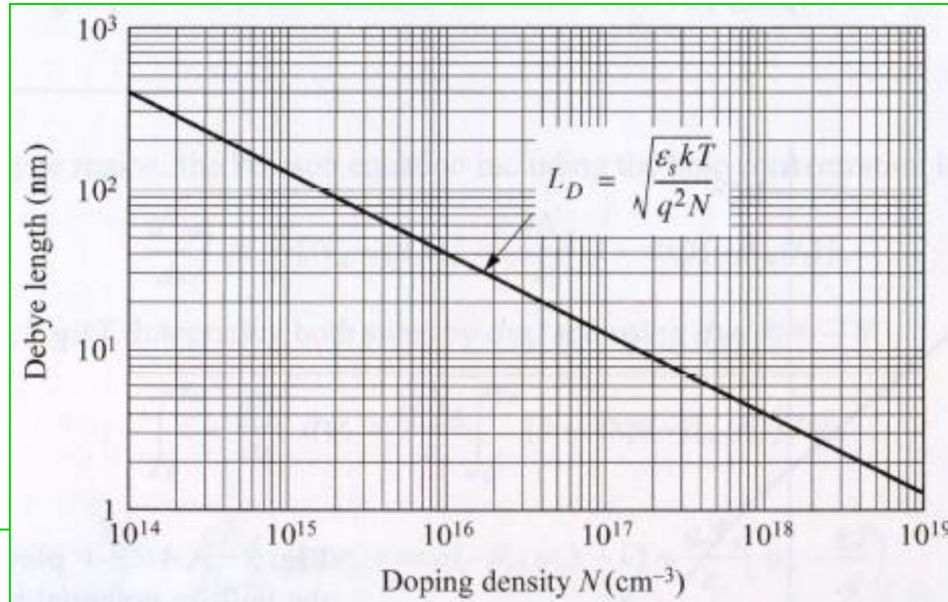


Fig.4 Debye length in Si at room temperature as a function of doping density *N*.