

# Theory of Semiconductor Devices (반도체 소자 이론)

## Lecture 20

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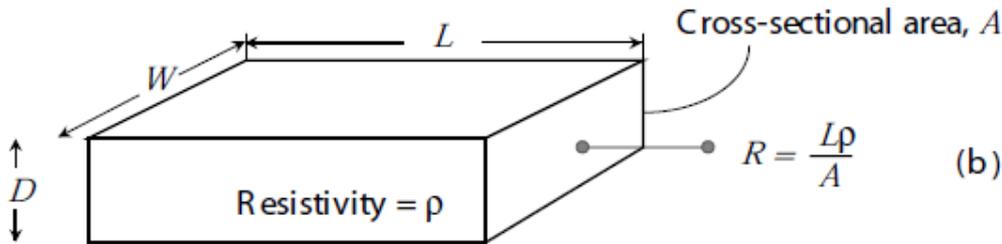
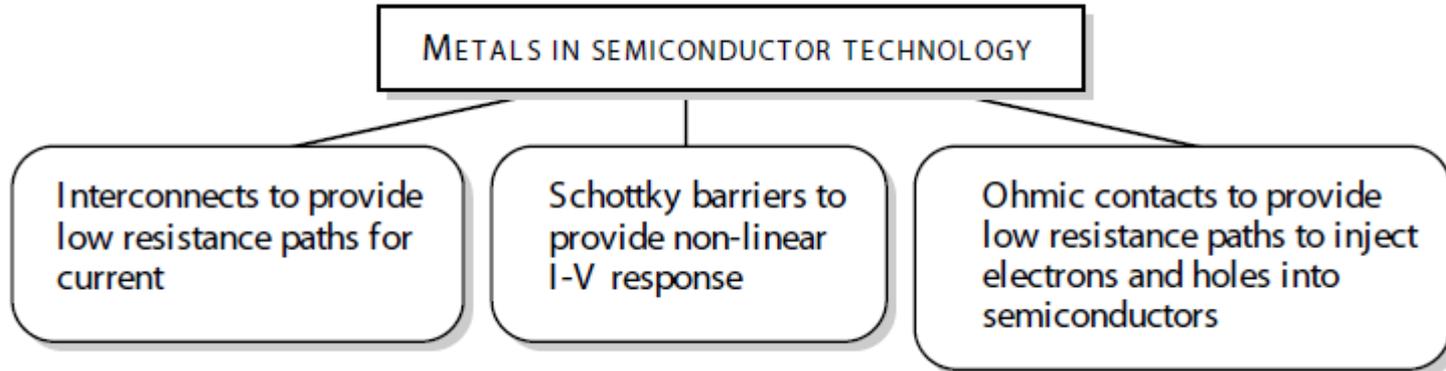
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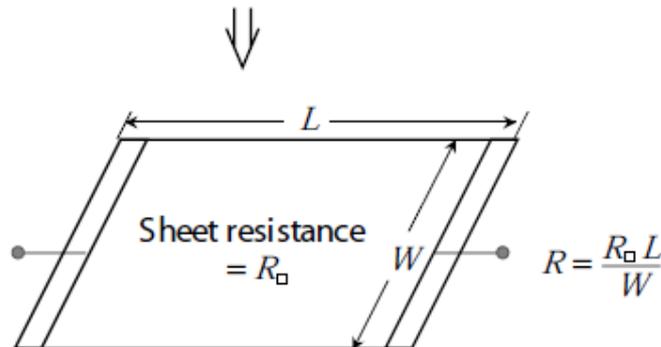
## Metal Semiconductor junction

# Metals in Semiconductor

Metals serve three important functions in semiconductor technology



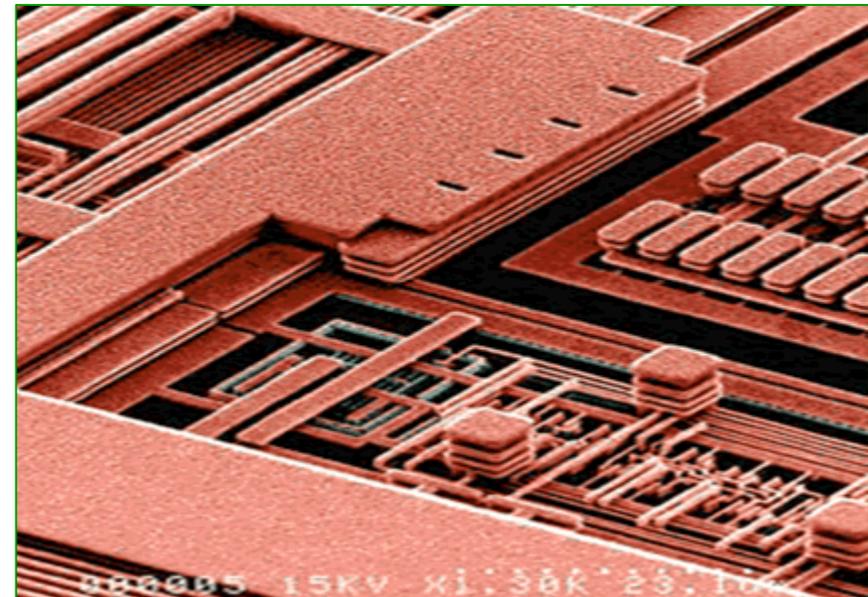
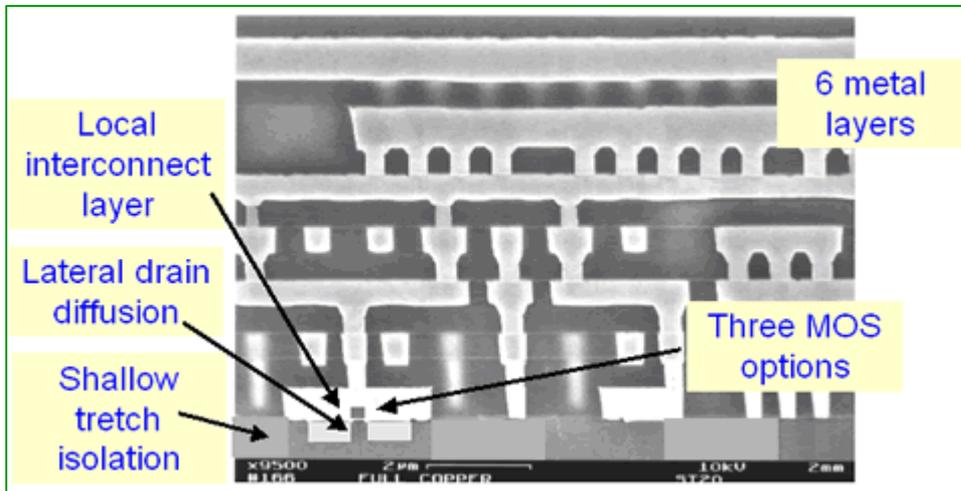
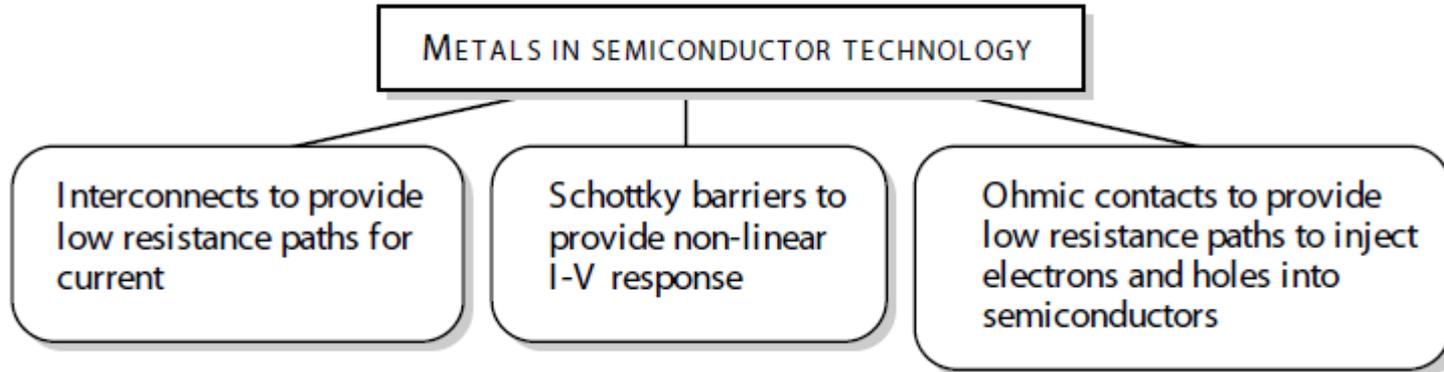
(b) A resistor of dimensions  $L \times W \times D$



(c) Representation of the resistors in terms of sheet resistance

# Metals in Semiconductor

Metals serve three important functions in semiconductor technology



# Metals in Semiconductor

## Example 5.1

In this example we will study some important concepts in thin-film resistors, which form an important part of semiconductor device technology. The resistors are often made from polysilicon that is appropriately doped. In thin-film technology it is usual to define sheet resistance instead of the resistance of the material. Consider, as shown in the previous figure (b), a material of length  $L$ , width  $W$ , and depth  $D$ . The **resistance** of the material is

$$R = \frac{\rho L}{WD} = \frac{\rho L}{A} \quad (5.2.1)$$

As we have studied previously, the **resistivity**  $\rho$  is given by

$$\rho = \frac{1}{ne\mu} \quad (5.2.2)$$

where  $n$  is the free **carrier density** and  $\mu$  is the **mobility** of the carriers (the equation can be modified for a p-type material).

The **sheet resistance** is a measure of the characteristics of a uniform sheet of film. It is defined as **ohms per square**, as shown in figure (c), and is related to the **film resistance** by

$$R_{\square} = R \frac{W}{L} \quad (5.2.3)$$

# Schottky Barrier

## METAL SEMICONDUCTOR JUNCTION: SCHOTTKY BARRIER

The metal-semiconductor junction can result in a junction that has **non-linear diode characteristics** similar to those of the  $p$ - $n$  diode except that for many applications it has a much **faster** response since **carrier transport is unipolar**. Such a junction is called a **Schottky barrier diode**.

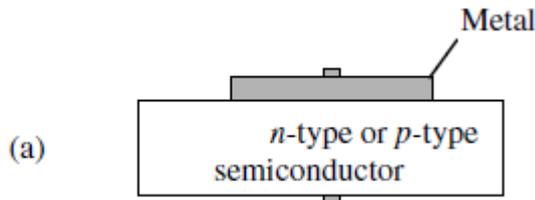
### Schottky Barrier Height

The working of the Schottky diode depends upon **how** the **metal-semiconductor junction** behaves in response to external bias. Let us pursue the *approximation* we used for the  $p$ - $n$  junction and examine the **band profile of a metal and a semiconductor**.

- A metal semiconductor structure is shown in figure 5.2a. In figure 5.2b and figure 5.2c the band profiles of a metal and a semiconductor are shown.
- Figure 5.2b shows that the band profile and Fermi level positions when the metal is *away* from the semiconductor.
- In figure 5.2c the metal and the semiconductor are *in contact*. The **Fermi level  $E_{Fm}$**  in the metal lies in the band, as shown.
- Also shown is the **work function  $e\phi_m$** . In the semiconductor, we show the **vacuum level** along with the position of the Fermi level  $E_{Fs}$  in the semiconductor, the **electron affinity**, and the **work function**.

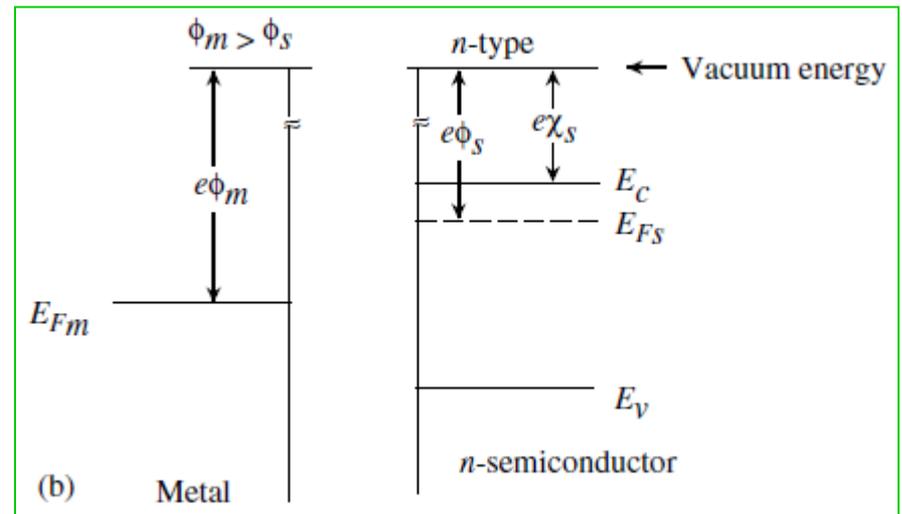
# Schottky Barrier

- We will *assume* an *ideal* surface for the semiconductor in the first calculation.
- Later we will examine the effect of *surface defects*. We will *assume* that  $\phi_m > \phi_s$  so that the Fermi level in the metal is at a lower position than in the semiconductor. This condition leads to an *n-type* Schottky barrier.



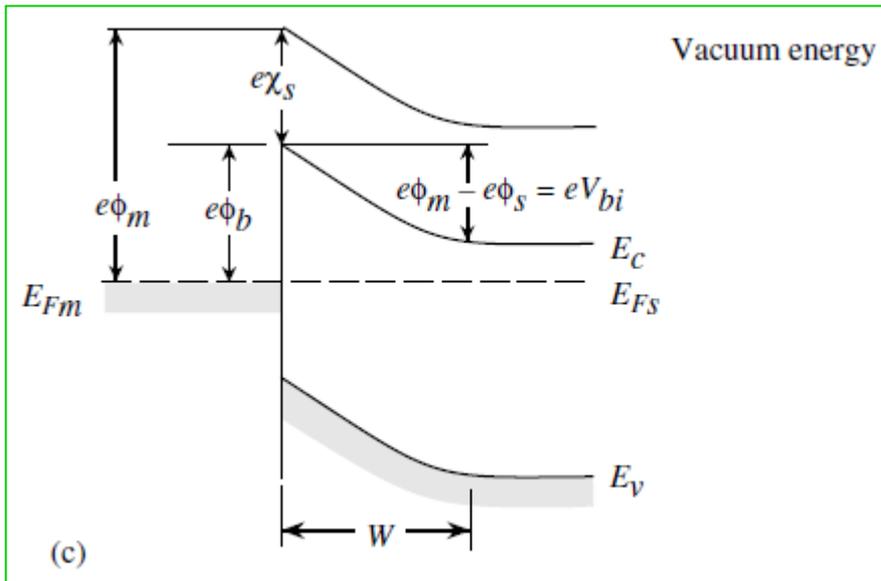
(a) A schematic of a metal-semiconductor junction

The various important energy levels in the metal and the semiconductor with respect to the vacuum level



# Schottky Barrier

- When the junction between the two systems is formed, the **Fermi levels should line up at the junction and remain flat in the absence of any current**, as shown in figure 5.2c.
- At the junction, the vacuum energy levels of the metal side and semiconductor side must be the same.
- **Electrons move out from the semiconductor side to the metal side.**
- Note that since the metal side has an enormous electron density, the metal Fermi level or the band profile does not change when a small fraction of electrons are added or taken out.
- As electrons move to the metal side, they leave behind positively charged fixed dopants, and a **dipole region is produced** in the same way as for the *p-n* diode.



The **junction potential** produced when the metal and semiconductor are **brought together**. Due to the **built-in potential** at the junction, a **depletion region** of width  $W$  is created

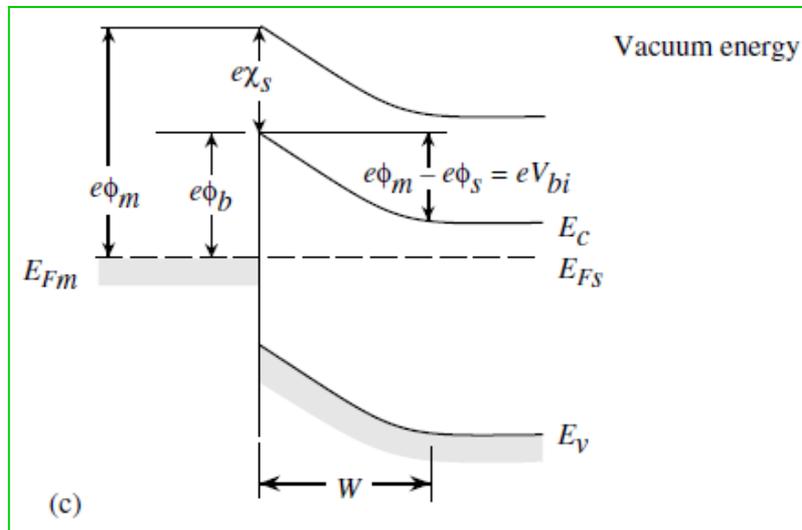
# Schottky Barrier

- In the *ideal* Schottky barrier with **no bandgap defect levels**, the height of the barrier at the semiconductor-metal junction (figure 5.2c), is defined as the **difference between the semiconductor conduction band at the junction and the metal Fermi level**. This barrier is given by (see figure 5.2c)

$$e\phi_b = e\phi_m - e\chi_s \quad (5.3.1)$$

- The electrons coming from the semiconductor into the metal face a **barrier** denoted by  $eV_{bi}$  as shown in figure 5.2c. The potential  $eV_{bi}$  is called the **built-in potential of the junction** and is given by

$$eV_{bi} = -(e\phi_m - e\phi_s) \quad (5.3.2)$$



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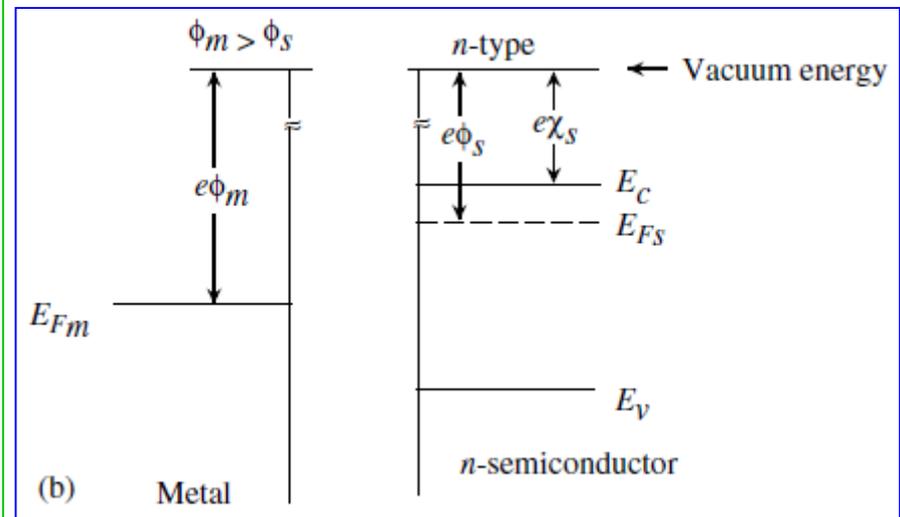
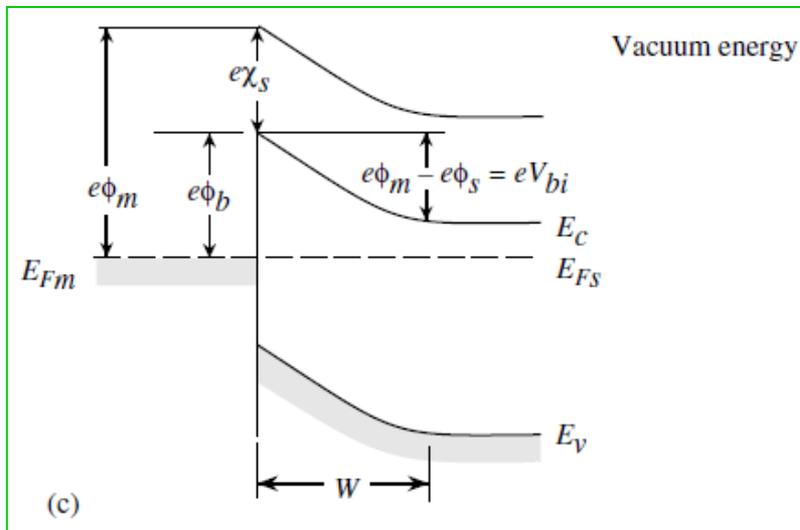
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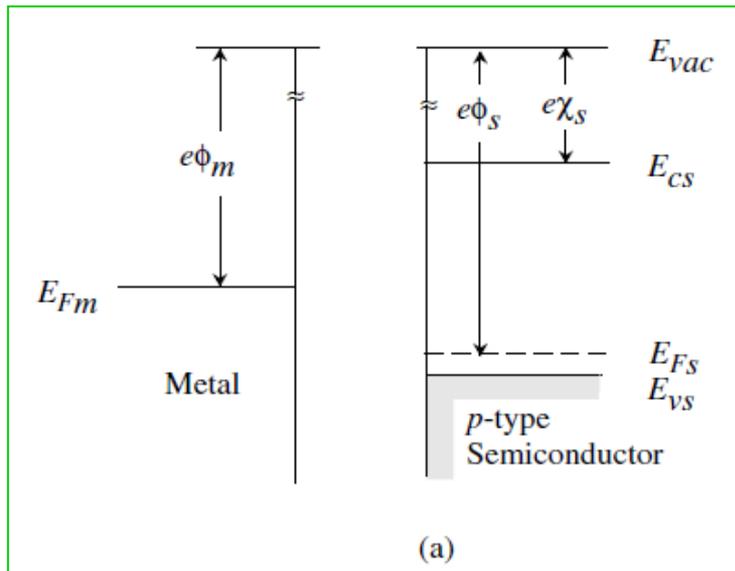


# Schottky Barrier

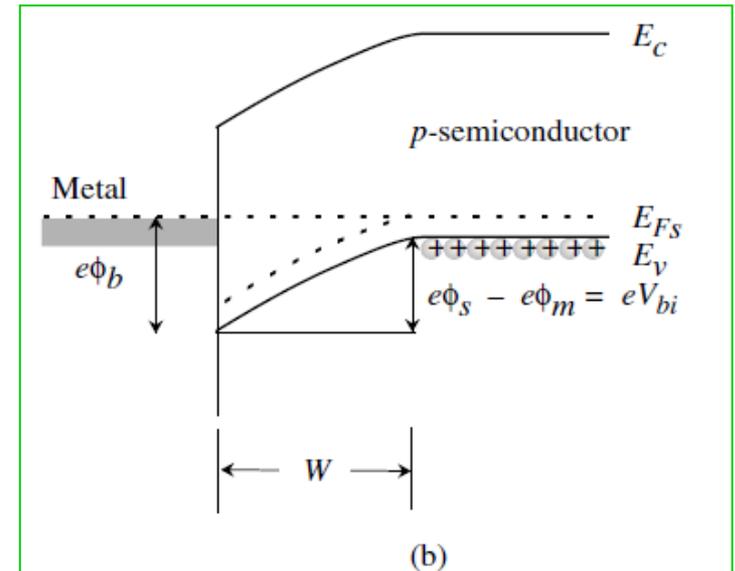
- It is possible to have a barrier for **hole transport** if  $\phi_m < \phi_s$ .
- In figure 5.3 we show the case of a metal - **p-type semiconductor** junction where we choose a metal so that  $\phi_m < \phi_s$ . In this case, at equilibrium the electrons are injected from the metal to the semiconductor, causing a negative charge on the semiconductor side.
- The bands are bent once again and a **barrier** is created for hole transport.
- The **height** of the barrier seen by the holes in the semiconductor is

$$eV_{bi} = e\phi_s - e\phi_m \quad (5.3.3)$$

A schematic of the ideal **p-type** Schottky barrier formation



The positions of the energy levels in the metal and the semiconductor



The junction potential and the depletion width

# Schottky Barrier

- The **Schottky barrier height** for *n*- or *p*-type semiconductors **depends** upon the metal and the **semiconductor properties**. This is **true** for an *ideal* case.
- It is found **experimentally** that the Schottky barrier height is **often independent of the metal employed**, as can be seen from table 5.2.

-This can be understood qualitatively in terms of a model based upon **non ideal surfaces**. In this model the metal-semiconductor interface has a distribution of **interface states** that may arise from the presence of **chemical defects** from **exposure to air** or **broken bonds**, etc.

- Defects can create bandgap states in a semiconductor.

- Surface defects can create  $\sim 10^{13} \text{ cm}^{-2}$  defects if there is **1 in 10 defects at the surface**.

SCHOTTKY METAL	<i>n</i> Si	<i>p</i> Si	<i>n</i> GaAs
Aluminum, Al	0.7	0.8	
Titanium, Ti	0.5	0.61	
Tungsten, W	0.67		
Gold, Au	0.79	0.25	0.9
Silver, Ag			0.88
Platinum, Pt			0.86
PtSi	0.85	0.2	
NiSi <sub>2</sub>	0.7	0.45	

Table 5.2: **Schottky barrier heights** (in volts) for several metals on *n*- and *p*-type semiconductors

# Schottky Barrier

- Surface defects lead to a **distribution of electronic levels in the bandgap at the interface**, as shown in figure 5.4. The distribution may be characterized by a **neutral level  $\phi_o$**  having the property that **states below it are neutral if filled** and **above it are neutral if empty**.
- If the density of bandgap states near  $\phi_o$  is very large, then addition or depletion of electrons to the semiconductor can not alter the Fermi level position at the surface without large changes in surface charges (beyond the numbers demanded by charge neutrality considerations). Thus, the Fermi level is said to be **pinned**.

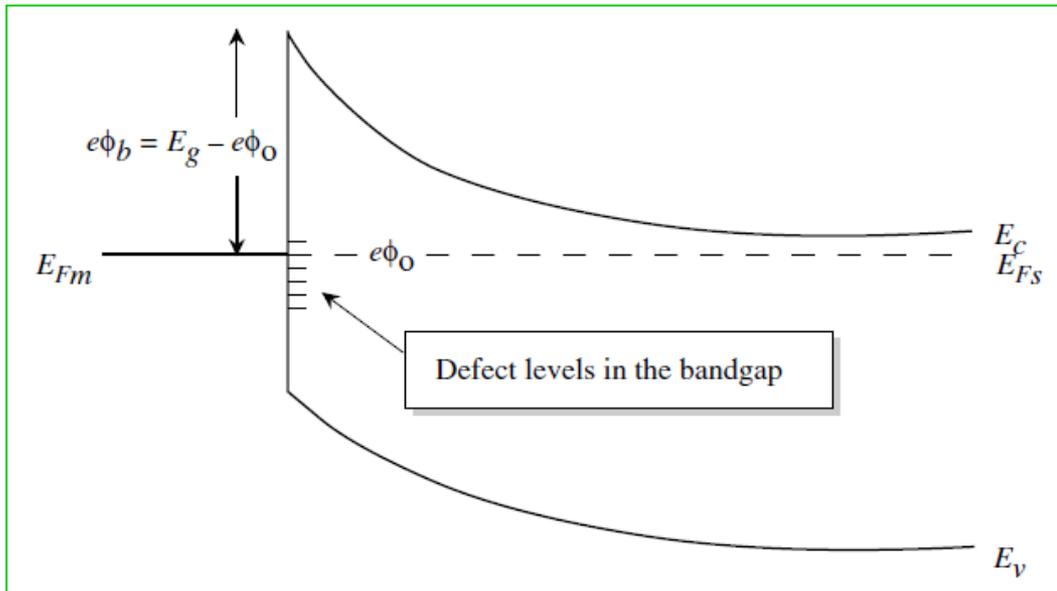


Figure 5.4: Interface states at a **real** metal-semiconductor interface. A **neutral level  $\phi_o$**  is defined so that the interface states above  $\phi_o$  are neutral if they are empty and those below  $\phi_o$  filled.

# Schottky Barrier

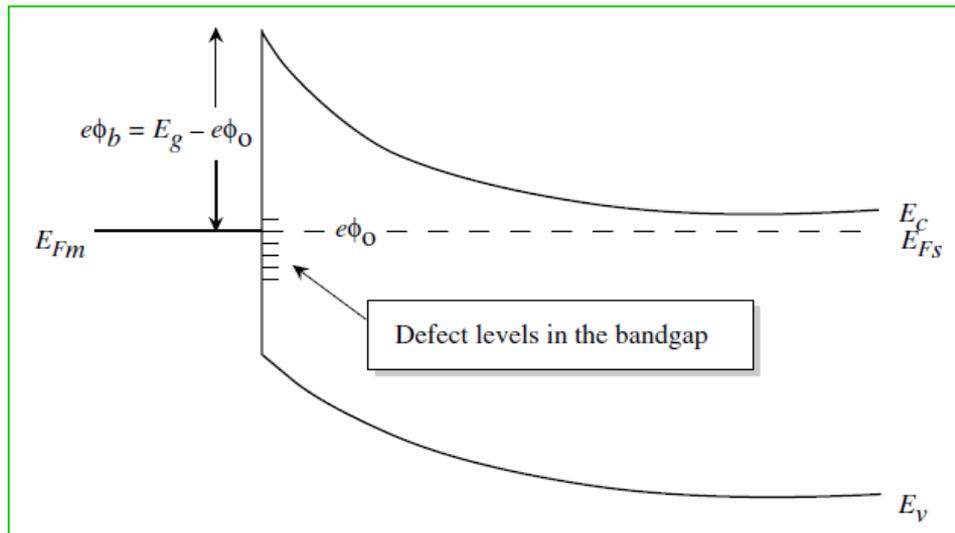
- In this case, as shown in figure 5.4, the Schottky barrier height is

$$e\phi_b = E_g - e\phi_o \quad (5.3.4)$$

and is almost independent of the metal used.

- The model discussed above provides a *qualitative* understanding of the Schottky barrier heights. However, the detailed mechanism of the interface state formation and *Fermi level pinning* is quite complex.

- In table 5.2 we show Schottky barrier heights for some common metal-semiconductor combinations. In some materials such as GaN and AlGaN, the surface retains its ideal behavior and the Schottky barrier is indeed controlled by the metal work function.



# Capacitance Voltage Characteristics

- Once the Schottky **barrier height** is known, the **electric field profile**, **depletion width**, **depletion capacitance**, etc., can be evaluated the same way we obtained the values for the *p-n* junction. The problem for a Schottky barrier on an *n*-type material is identical to that for the abrupt *p<sup>+</sup>n* diode, since there is **no depletion on the metal side**. One again makes the depletion approximation; i.e., there is no mobile charge in the depletion region and the semiconductor is neutral outside the depletion region.
- Then the solution of the Poisson equation gives the depletion width *W* for an external voltage applied to the metal *V*

$$W = \left[ \frac{2\epsilon(V_{bi} - V)}{eN_d} \right]^{1/2} \quad (5.3.5)$$

Here  $N_d$  is the doping of the *n*-type semiconductor.

- Note that there is no depletion on the metal side because of the high electron density there. The potential *V* is the applied potential, which is positive for forward bias and negative for reverse bias.

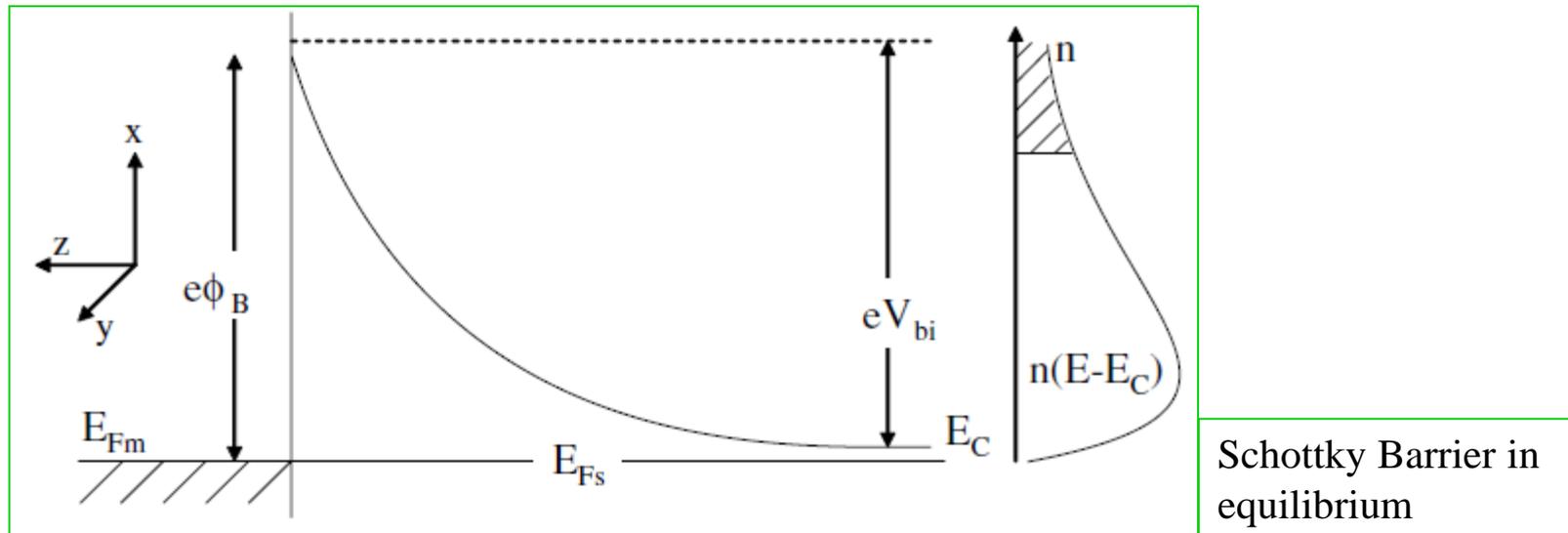
# Thermionic Emission

## \* Current Flow across a Schottky Barrier: Thermionic Emission

- Consider the Schottky barrier band diagram shown below.
- The Schottky barrier between a metal and semiconductor is shown in **equilibrium** (at zero bias) with the **electron distribution** shown on the right.
- The electron distribution:

$$n(E - E_C) = 2f(E - E_C) \cdot N(E - E_C) \quad (5.3.6)$$

similar to the case of a  $p-n$  junction, the factor of 2 in accounting for electron spin.



# Thermionic Emission

- **Thermionic emission** *assumes* that all electrons in the semiconductor with kinetic energy in the  $+z$  direction greater than  $eV_{bi}$  ( $E_z > eV_{bi}$ ) and  $k_z > 0$ , are **capable of surmounting the barrier** and **contributing to current flow** from the semiconductor to the metal,  $J_{s \rightarrow m}$ .
- Note that the total kinetic energy  $E - E_C = E_x + E_y + E_z$ .
- At **thermal equilibrium** the current from the metal to the semiconductor,  $J_{m \rightarrow s}$ , will be **equal** in magnitude and opposite in sign to  $J_{s \rightarrow m}$ , making the **net current zero**.
- To calculate  $J_{s \rightarrow m}$  one needs to sum the current carried by every allowed electron:

$$J_{s \rightarrow m} = e \sum n(E - E_C) \cdot v_z \quad (5.3.7)$$

for  $E_z > eV_{bi}$  and  $v_z > 0$ .

- The methodology employed is to calculate the number of electrons at energy  $E$  in a volume of  $k$ -space  $(dk)^3$ , multiply the number with the electron velocity in the direction along the barrier, and sum or integrate over energy.
- Assuming a crystal of length  $L$ , **periodic boundary conditions** yield allowed  $k$  values given by

$$k = 2\pi N \quad (5.3.8)$$

where  $N$  is an integer and the separation between allowed  $k$ 's is  $\Delta k = 2\pi/L$ .

# Thermionic Emission

- The **number of electrons** in a volume element  $dk_x, dk_y, dk_z$  is therefore

$$dN = 2f(E - E_C) \frac{dk_x dk_y dk_z}{\Delta k^3} \quad (5.3.9)$$

Assuming  $(E - E_C) \gg E_F$  and writing  $E - E_F = E - E_C + E_C - E_F$  gives

$$dN = 2 \exp\left(\frac{-((E - E_C) + (E_C - E_F))}{k_B T}\right) \frac{dk_x dk_y dk_z}{\Delta k^3} \quad (5.3.10)$$

- The **current density** contributed by these electrons is

$$J_z = -ev_z \frac{dN}{L^3} \quad (5.3.11)$$

if  $k_z > 0$  and  $E_z > eV_{bi}$ . Note that all values of  $E_x$  and  $E_y$  are allowed as they represent motion in the  $x - y$  plane which is not constrained by the barrier in the  $+z$  direction. Note that

$$(E_x - E_C) = \frac{\hbar^2 k_x^2}{2m^*} \quad (5.3.12)$$

with similar relationships for  $(E_y - E_C)$  and  $(E_z - E_C)$ .

# Thermionic Emission

- Also employing the condition  $(E_z - E_C) > E_z > eV_{bi}$  yields a minimum value of

$$k_{min} = \sqrt{eV_{bi} \left( \frac{2m^*}{\hbar^2} \right)} \quad (5.3.13)$$

Also

$$v_z = \frac{\hbar k_z}{m^*} \quad (5.3.14)$$

Therefore,

$$\begin{aligned}
 J_z &= \frac{-e}{(2\pi)^3} \int_{-\infty}^{+\infty} dk_x \int_{-\infty}^{+\infty} dk_y \int_{k_{min}}^{+\infty} \frac{\hbar k_z}{m^*} dk_z \cdot \\
 &2 \exp[-(E_x + E_y + E_z)/k_B T] \cdot \exp[-(E_C - E_F)/k_B T] \exp\left(\frac{E_C}{k_B T}\right) \\
 &= -\frac{2e}{(2\pi)^3} \int_x \cdot \int_y \cdot \int_z \exp\left(-\frac{E_C - E_F}{k_B T}\right)
 \end{aligned} \quad (5.3.15)$$

where

$$\int_x = \int_y = \int_{-\infty}^{\infty} \exp\left(\frac{\hbar^2 k_x^2}{2m^* k_B T}\right) dk_x = \frac{\sqrt{2\pi m^* k_B T}}{\hbar} \quad (5.3.16)$$

# Thermionic Emission

and

$$\int_z = \int_{k_{min}}^{\infty} \exp\left(-\frac{\hbar^2 k_z^2}{k_B T}\right) \cdot \frac{\hbar k_z}{m^*} \cdot dk_z \quad (5.3.17)$$

$$= \frac{k_B T}{\hbar} \exp(-\hbar^2 k_{min}^2 / k_B T) = \frac{k_B T}{\hbar} \exp\left(\frac{-eV_{bi}}{k_B T}\right) \quad (5.3.18)$$

Therefore,

$$J_z = \frac{4\pi}{(2\pi\hbar)^3} \cdot em^* k_B^2 T^2 \exp\left(-\frac{(eV_{bi} + (E_C - E_F))}{k_B T}\right) \quad (5.3.19)$$

or

$$J_z = A^* \cdot T^2 \exp\left(\frac{-e\phi_B}{k_B T}\right) = J_{s \rightarrow m} (V = 0) \quad (5.3.20)$$

where

$$A^* = \frac{4\pi em^* k_B^2}{2\pi\hbar^3} = 120 \text{ A cm}^{-2} \text{ K}^{-2} \times \frac{m^*}{m_0} \quad (5.3.21)$$

is the [Richardson constant](#) and  $\phi_B = V_{bi} + (E_C - E_F)$ , the barrier seen by electrons in the metal of the Schottky barrier height.

# Thermionic Emission

- We have calculated  $J_{s \rightarrow m}$  at  $V = 0$ .
- The analysis can be easily extended to a forward bias of  $V_F$ , the only change being replacing the barrier,  $V_{bi}$  by the new barrier  $V_{bi} - V_F$ . This changes  $I_z$  to

$$I_z = \frac{k_B T}{\hbar} \exp\left(-\frac{eV_{bi}}{k_B T}\right) \cdot \exp\left(\frac{eV_F}{k_B T}\right) \quad (5.3.22)$$

or

$$J_{s \rightarrow m}(V = V_F) = J_{s \rightarrow m}(V = 0) \cdot \exp\left(\frac{eV_F}{k_B T}\right) \quad (5.3.23)$$

Since the current flow from the metal to the semiconductor is **unchanged**:

$$J(V = V_F) = J_{s \rightarrow m}(V = V_F) - J_{m \rightarrow s}(V = V_F) \quad (5.3.24)$$

$$= A^* T^2 \exp\left(\frac{-q\phi_B}{k_B T}\right) \left[ \exp\left(\frac{eV_F}{k_B T}\right) - 1 \right] \quad (5.3.25)$$

# Thermionic Emission: Example

**Example 5.2** In a  $W-n$ -type Si Schottky barrier the semiconductor has a doping of  $10^{16} \text{ cm}^{-3}$  and an area of  $10^{-3} \text{ cm}^2$ .

(a) Calculate the 300 K diode current at a forward bias of 0.3 V.

(b) Consider an Si  $p^+ - n$  junction diode with the same area with doping of  $N_a = 10^{19} \text{ cm}^{-3}$  and  $N_d = 10^{16} \text{ cm}^{-3}$ , and  $\tau_p = \tau_n = 10^{-6} \text{ s}$ . At what forward bias will the  $p-n$  diode have the same current as the Schottky diode?  $D_p = 10.5 \text{ cm}^2/\text{s}$ .

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$$J_z = A^* \cdot T^2 \exp\left(\frac{-e\phi_B}{k_B T}\right)$$

From table 5.2 the Schottky barrier of  $W$  on Si is 0.67 V. Using an effective Richardson constant of  $110 \text{ A cm}^{-2} \text{ K}^{-1}$ , we get for the reverse saturation current

$$\begin{aligned} I_s &= (10^{-3} \text{ cm}^2) \times (110 \text{ A cm}^{-2} \text{ K}^{-2}) \times (300 \text{ K})^2 \exp\left(\frac{-0.67(\text{eV})}{0.026(\text{eV})}\right) \\ &= 6.37 \times 10^{-8} \text{ A} \end{aligned}$$

# Thermionic Emission: Example

$$A^*T^2 \exp\left(\frac{-q\phi_B}{k_B T}\right) \left[ \exp\left(\frac{eV_F}{k_B T}\right) - 1 \right]$$

For a forward bias of 0.3 V, the current becomes (neglecting 1 in comparison to  $\exp(0.3/0.026)$ )

$$\begin{aligned} I &= 6.37 \times 10^{-8} \text{ A} \exp(0.3/0.026) \\ &= 6.53 \times 10^{-3} \text{ A} \end{aligned}$$

In the case of the  $p$ - $n$  diode, we need to know the appropriate diffusion coefficients and lengths. The diffusion coefficient is  $10.5 \text{ cm}^2/\text{s}$ , and using a value of  $\tau_p = 10^{-6} \text{ s}$  we get  $L_p = 3.24 \times 10^{-3} \text{ cm}$ . Using the results for the abrupt  $p^+ - n$  junction, we get for the saturation current ( $p_n = 2.2 \times 10^4 \text{ cm}^{-3}$ ) (note that the saturation current is essentially due to hole injection into the  $n$ -side for a  $p^+ - n$  diode)

$$\begin{aligned} I_o &= (10^{-3} \text{ cm}^2) \times (1.6 \times 10^{-19} \text{ C}) \times \frac{(10.5 \text{ cm}^2/\text{s}^{-1})}{(3.24 \times 10^{-3} \text{ cm})} \times (2.25 \times 10^4 \text{ cm}^{-3}) \\ &= 1.17 \times 10^{-14} \text{ A} \end{aligned}$$

$$J_p = -qD_p \frac{dp_n}{dx} \Big|_{w_{Dn}} = \frac{qD_p p_{no}}{L_p} \left[ \exp\left(\frac{qV}{kT}\right) - 1 \right]. \quad (62a)$$

# Thermionic Emission: Example

This is an extremely small value of the current. At 0.3 V, the diode current becomes

$$I = I_s \exp \left( \frac{eV}{k_B T} \right) = 1.2 \times 10^{-9} \text{ A}$$

a value which is almost six orders of magnitude smaller than the value in the Schottky diode. For the  $p-n$  diode to have the same current that the Schottky diode has at 0.3 V, the voltage required is 0.71 V.

This example highlights the important differences between Schottky and junction diodes.  
The Schottky diode turns on (i.e., the current is  $\sim 1$  mA) at 0.3 V while the  $p-n$  diode turns on at closer to 0.7 V.

# Schottky vs $p-n$ diodes

- Both the  $p-n$  diode and the Schottky diode can be used for rectification and non-linear  $I-V$  response.

$p-n$ DIODE	SCHOTTKY DIODE
Reverse current is very low	Reverse current is relatively large
Forward current due to minority carrier injection from $n$ - and $p$ -sides	Forward current due to majority injection from the semiconductor
Forward bias needed to make the device conducting (the cut-in voltage) is large	The cut-in voltage is quite small
Switching speed controlled by recombination (elimination) of minority injected carriers	Device very fast: switching speed controlled by thermalization of "hot" injected electrons across the barrier $\sim$ few picoseconds
Ideality factor in $I-V$ characteristics $\sim 1.2-2.0$ due to recombination in depletion region	Essentially no recombination in depletion region $\rightarrow$ ideality factor $\sim 1.0$

# Ohmic Contacts

- It is possible to create metal-semiconductor junctions that have a **linear non-rectifying I-V characteristic**, as shown in figure 5.7. Such junctions or contacts are called **ohmic contacts**.
- There are **two possibilities** for creating ohmic contacts. In the previous section, to produce a Schottky barrier on an n-type semiconductor, we needed (for the ideal surface) a metal with a work function larger than that of the semiconductor.
- Thus, **in principle**, if we use a **metal with a work function smaller than the semiconductor**, one should have **no built-in barrier**. However, this approach is not often useful in practice because the Fermi level at the surface of real semiconductors is pinned because of the high interface density in the gap.
- The Schottky barrier discussed earlier can be altered to create an ohmic contact. This is done **through heavy doping and use of tunneling to get large current across the interface**.
- Let us say we have a built-in potential barrier,  $V_{bi}$ . The depletion width on the semiconductor side is

$$W = \left[ \frac{2\epsilon V_{bi}}{eN_d} \right]^{1/2} \quad (5.4.1)$$

# Ohmic Contacts

- Now if *near* the interface region the semiconductor is **heavily doped**, the **depletion width** could be made **extremely narrow**. In fact, it can be made so narrow that even though there is a potential barrier, the electrons can tunnel through the barrier with ease, as shown in figure 5.7.

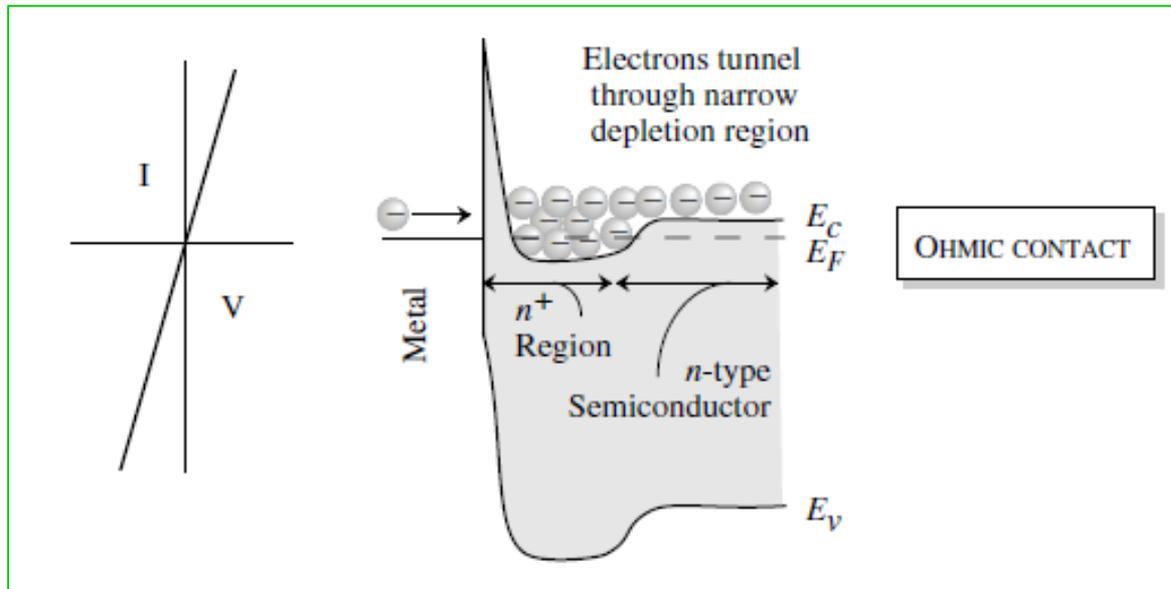


Figure 5.7: Current-voltage characteristics of an **ohmic contact** along with the band diagrams of metal -  $n^+$ - $n$  semiconductor contact. The **heavy doping** reduces the depletion width to such an extent that the electrons can **tunnel** through the spiked barrier easily in either direction.

# Ohmic Contacts

- The **quality** of an ohmic contact is usually **defined** through the **resistance R** of the contact over a certain **area A**. The normalized resistance is called the **specific contact resistance  $r_c$**  and is given by

$$r_c = R \cdot A \quad (5.4.2)$$

- Under conditions of **heavy doping** where the transport is by **tunneling**, the **specific contact resistance** has the following dependence for tunneling, probability  $T$ , through a triangular barrier):

$$\ln(r_c) \propto \frac{1}{\ln(T)} \propto \frac{(V_{bi})^{3/2}}{F} \quad (5.4.3)$$

where the field is

$$\mathcal{E} = \frac{V_{bi}}{W} \propto (V_{bi})^{1/2} (N_d)^{1/2} \quad (5.4.4)$$

Thus,

$$\ln(r_c) \propto V_{bi} \propto \frac{1}{\sqrt{N_d}} \quad (5.4.5)$$

- The resistance can be reduced by using a low Schottky barrier height and doping as heavily as possible. The predicted dependence of the contact resistance on the doping density is, indeed, observed experimentally.