Theory of Semiconductor Devices
(반도체 소자 이론)

Lecture 9. The PN diodes

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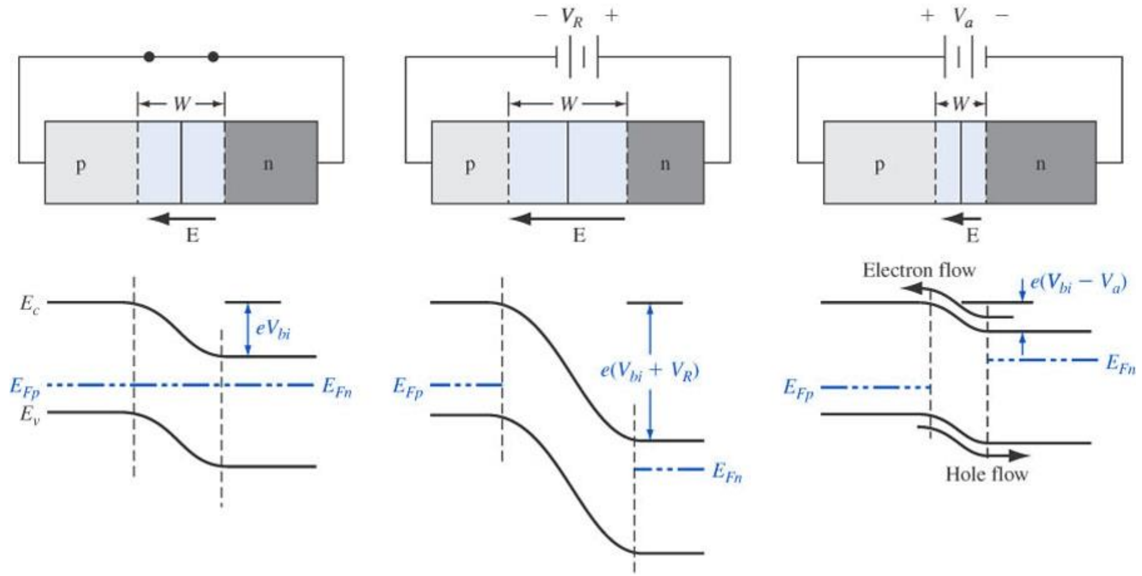
Qualitative Description of Charge Flow in a pn Junction

- Potential barrier for electrons and holes :

  Equilibrium: eV_{bi}

  Reverse biased: $e(V_{bi} + V_R)$

  Forward biased: $e(V_{bi} - V_a)$



The smaller potential barrier means that the electric field in the depletion region is also reduced.

The smaller electric field means that the electrons and holes are no longer held back in the n and p region, respectively.

→ There will be a diffusion of holes from the p region (and a diffusion of electrons from n region)

→ The flow of charge generates a current through the pn junction

The injected holes/electrons into the n/p region : acts as excess minority carriers.

→ The behavior of these minority carriers is described by the ambipolar transport equations.

→ There will be diffusion as well as recombination of excess carriers in these regions.

Qualitative Description of Charge Flow in a pn Junction

Ideal Current-Voltage Relationship

- ✓ Abrupt depletion layer approximation
- ✓ Maxwell-Boltzmann approximation
- ✓ Low injection
- ✓ Total current is a constant. Electron and hole currents are continuous functions through the pn structures and constant throughout the depletion region.

Table 8.1 | Commonly used terms and notation for this chapter

Term	Meaning
N_a	Acceptor concentration in the p region of the pn junction
N_d	Donor concentration in the n region of the pn junction
$n_{n0} = N_d$	Thermal-equilibrium majority carrier electron concentration in the n region
$p_{p0} = N_a$	Thermal-equilibrium majority carrier hole concentration in the p region
$n_{p0} = n_i^2/N_a$	Thermal-equilibrium minority carrier electron concentration in the p region
$p_{n0} = n_i^2/N_d$	Thermal-equilibrium minority carrier hole concentration in the n region
n_p	Total minority carrier electron concentration in the p region
p_n	Total minority carrier hole concentration in the n region
$n_p(-x_p)$	Minority carrier electron concentration in the p region at the space charge edge
$p_n(x_n)$	Minority carrier hole concentration in the n region at the space charge edge
$\delta n_p = n_p - n_{p0}$	Excess minority carrier electron concentration in the p region
$\delta p_n = p_n - p_{n0}$	Excess minority carrier hole concentration in the n region

Boundary Conditions

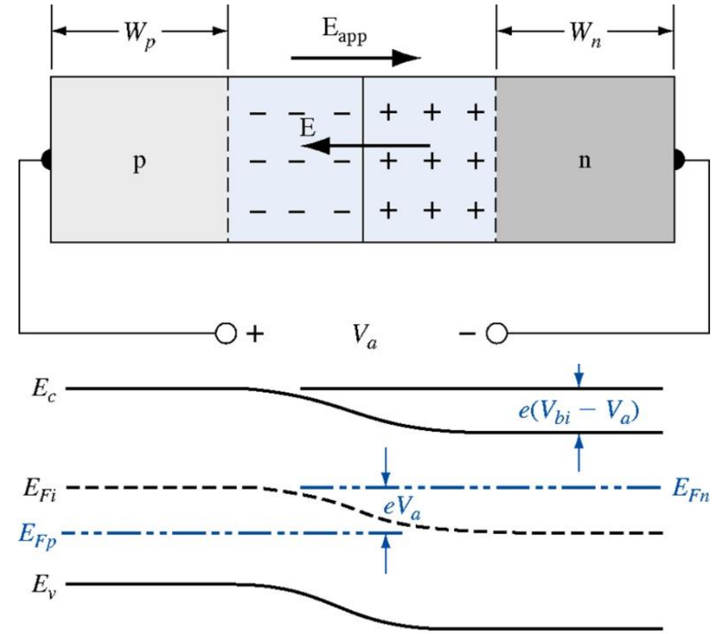
$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right) \Rightarrow \frac{n_i^2}{N_a N_d} = \exp\left(\frac{-eV_{bi}}{kT}\right)$$

$$n_{n0} \approx N_d$$

$$n_{p0} \approx \frac{n_i^2}{N_a}$$

$$n_{p0} = n_{n0} \exp\left(\frac{-eV_{bi}}{kT}\right)$$

: define the relation between the electrons in n-type and p-type regions



For a forward bias condition, $V_{bi} \rightarrow (V_{bi} - V_a)$:

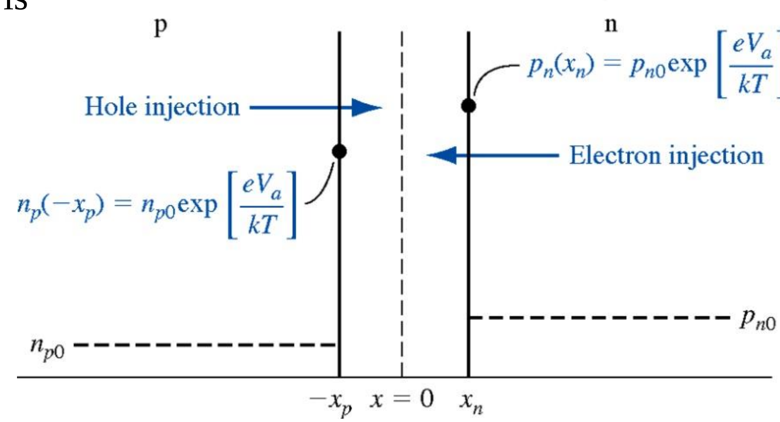
$$n_p = n_{n0} \exp\left(\frac{-e(V_{bi} - V_a)}{kT}\right) = n_{n0} \exp\left(\frac{-eV_{bi}}{kT}\right) \exp\left(\frac{+eV_a}{kT}\right)$$

$$\Rightarrow \quad n_p = n_{p0} \exp\left(\frac{eV_a}{kT}\right)$$

: the minority electron concentration in p-type region edge is increased from its thermal equilibrium value.

Similarly, the minority hole concentration in n-type region edge is increased from its thermal equilibrium value :

$$\Rightarrow \quad p_n = p_{n0} \exp\left(\frac{eV_a}{kT}\right)$$

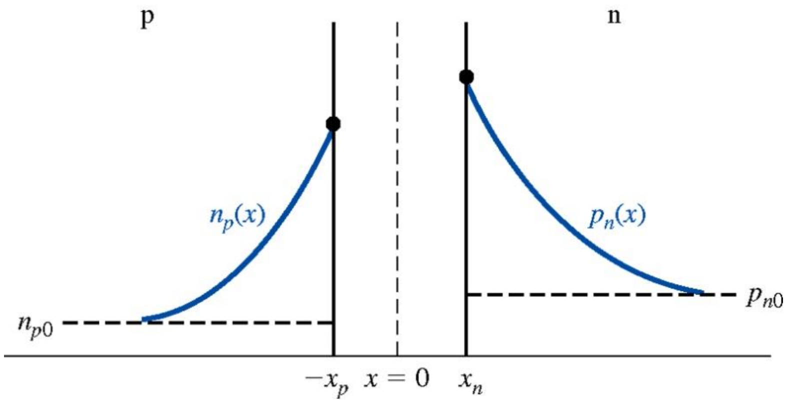


Example 8.1 : A silicon pn junction at T = 300K with $N_d = 10^{16}$ cm^{-3} and a forward bias of 0.6 V is applied to the junction. Calculate the minority carrier hole concentration at the edge of the space charge region.

$$p_{n0} = \frac{n_i^2}{N_d} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4 \text{ cm}^{-3}$$

$$p_n = p_{n0} \exp\left(\frac{eV_a}{kT}\right) = 2.25 \times 10^4 \exp\left(\frac{0.60}{0.0259}\right) = 2.59 \times 10^{14} \text{ cm}^{-3}$$

Minority Carrier Distribution



Ambipolar transport equation for excess minority holes in n region :

$$\Rightarrow \quad D_p \frac{\partial^2(\delta p_n)}{\partial x^2} - \mu_p \mathrm{E} \frac{\partial(\delta p_n)}{\partial x} + g' - \frac{\delta p_n}{\tau_{p0}} = \frac{\partial(\delta p_n)}{\partial t}$$

In n region for $x > x_n$, $E = 0$ and $g' = 0$, also assume steady state :

$$\Rightarrow \quad \frac{d^2(\delta p_n)}{dx^2} - \frac{\delta p_n}{L_p^2} = 0 \qquad (x > x_n) \qquad\qquad \frac{d^2(\delta n_p)}{dx^2} - \frac{\delta n_p}{L_n^2} = 0 \qquad (x < x_p)$$

General solutions :

$$\delta p_n(x) = p_n(x) - p_{n0} = Ae^{x/L_p} + Be^{-x/L_p} \qquad (x \geq x_n)$$

$$\delta n_p(x) = n_p(x) - n_{p0} = Ce^{x/L_n} + De^{-x/L_n} \qquad (x \leq -x_p)$$

Boundary conditions :

$$p_n(x_n) = p_{n0} \exp\left(\frac{eV_a}{kT}\right)$$

$$n_p(-x_p) = n_{p0} \exp\left(\frac{eV_a}{kT}\right)$$

$$p_n(x \to +\infty) = p_{n0}$$

$$n_p(x \to -\infty) = n_{p0}$$

$$\Rightarrow \quad \delta p_n(x) = p_n(x) - p_{n0} = p_{n0}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right] \exp\left(\frac{x_n - x}{L_p}\right)$$

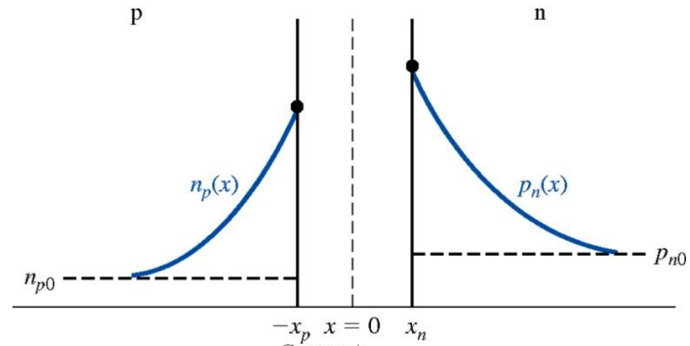
$$\delta n_p(x) = n_p(x) - n_{p0} = n_{p0}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right] \exp\left(\frac{x_p + x}{L_n}\right)$$

Ideal pn Junction Current

Total current : the minority carrier hole diffusion current at $x = x_n$

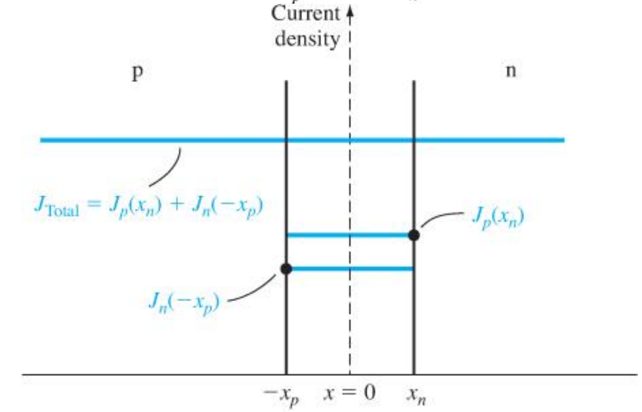
+ the minority carrier electron diffusion current at $x = -x_p$.

(neglect the minority carrier drift current at the neutral region)



$$J_p(x_n) = -eD_p \frac{dp_n(x)}{dx}\bigg|_{x=x_n} = -eD_p \frac{d(\delta p_n(x))}{dx}\bigg|_{x=x_n}$$

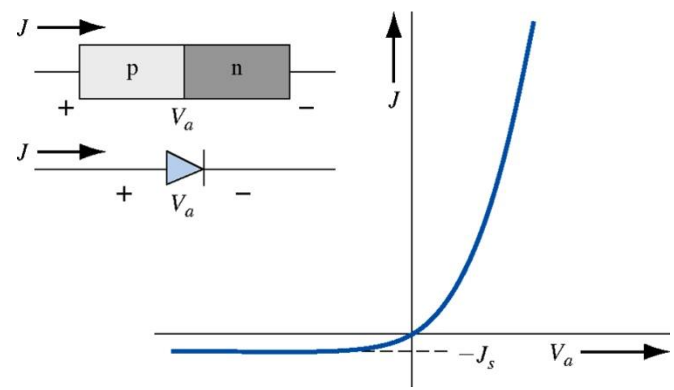
$$J_p(x_n) = \frac{eD_p p_{n0}}{L_p}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$



Similarly, electron diffusion current :

$$J_n(-x_p) = eD_n \frac{d(\delta n_p(x))}{dx}\bigg|_{x=-x_p} = \frac{eD_n n_{p0}}{L_n}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

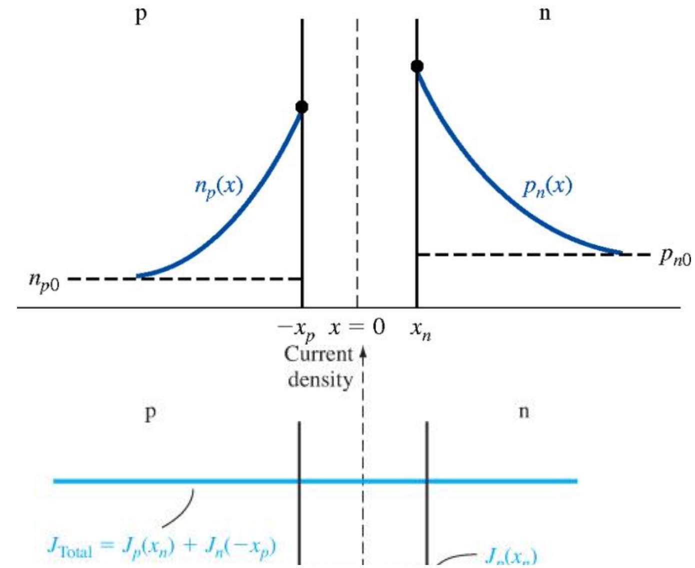
Each electron and hole currents are continuous and constant through the depletion region.



$$J = J_p(x_n) + J_n(-x_p) = \left[\frac{eD_p p_{n0}}{L_p} + \frac{eD_n n_{p0}}{L_n}\right]\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

$$= J_s\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right] \qquad where, \quad J_s = \left[\frac{eD_p p_{n0}}{L_p} + \frac{eD_n n_{p0}}{L_n}\right]$$

Ideal pn Junction Current



Total current : the minority carrier hole diffusion current at $x = x_n$

+ the minority carrier electron diffusion current at $x = -x_p$.

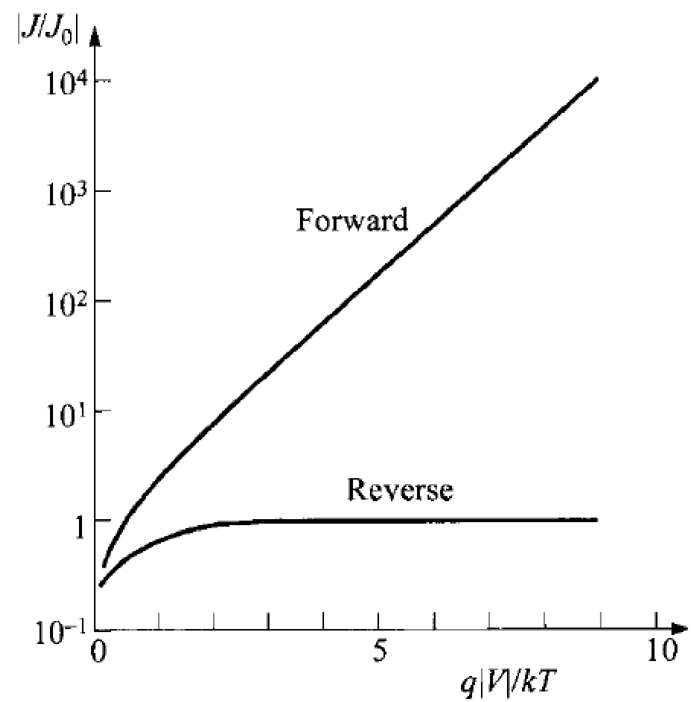
(neglect the minority carrier drift current at the neutral region)

$$J_p(x_n) = -eD_p \frac{dp_n(x)}{dx}\bigg|_{x=x_n} = -eD_p \frac{d(\delta p_n(x))}{dx}\bigg|_{x=x_n}$$

$$J_p(x_n) = \frac{eD_p p_{n0}}{L_p}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

Similarly, electron diffusion current :

$$J_n(-x_p) = eD_n \frac{d(\delta n_p(x))}{dx}\bigg|_{x=-x_p} = \frac{eD_n n_{p0}}{L_n}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$



Each electron and hole currents are continuous and constant through the depletion region.

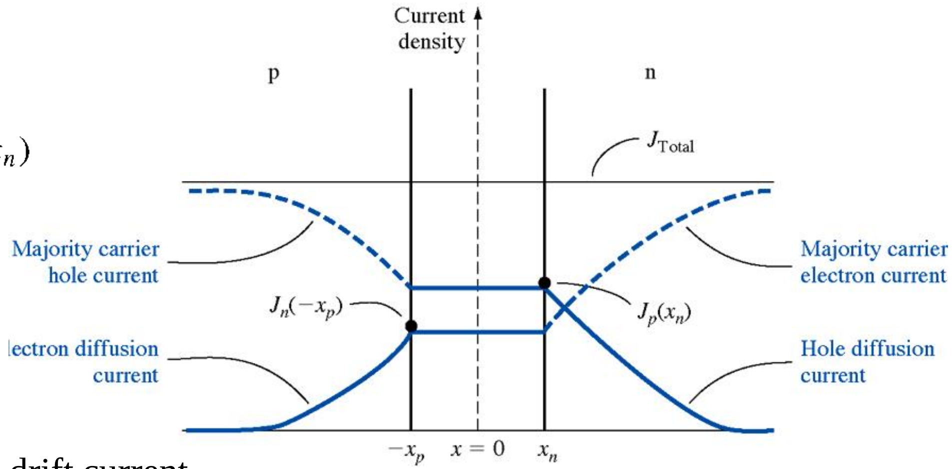
$$J = J_p(x_n) + J_n(-x_p) = \left[\frac{eD_p p_{n0}}{L_p} + \frac{eD_n n_{p0}}{L_n}\right]\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

$$= J_s\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right] \qquad where, \quad J_s = \left[\frac{eD_p p_{n0}}{L_p} + \frac{eD_n n_{p0}}{L_n}\right]$$

Minority carrier diffusion current densities :

$$J_p(x) = \frac{eD_p p_{n0}}{L_p}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]\exp\left(\frac{x_n - x}{L_p}\right) \qquad (x \ge x_n)$$

$$J_n(x) = \frac{eD_n n_{p0}}{L_n}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]\exp\left(\frac{x_p + x}{L_n}\right) \qquad (x \le -x_p)$$



Neutral p region (far away from junction) : mostly hole drift current
Neutral p region (near SCR) : minority electron diffusion current + majority hole drift current

Neutral n region (far away from junction) : mostly electron drift current
Neutral n region (near SCR) : minority hole diffusion current + majority electron drift current

Space Charge Region : electron and hole diffusion currents

Example 8.4 : A silicon pn junction at T = 300K with J_s = 4.15x10^{-11} A/cm^2 , N_d = 10^{16} cm^{-3} and V_a = 0.65 V. What is the electric field to produce a given majority carrier drift current ?

$$J = (4.15 \times 10^{-11})\left[\exp\left(\frac{0.65}{0.0259}\right) - 1\right] = 3.29 \text{ A/cm}^2$$

At neutral n region far away from SCR :

$$J = J_n \approx e\mu_n N_d \mathrm{E}$$

$$\mathrm{E} = \frac{J_n}{e\mu_n N_d} = \frac{3.29}{(1.6 \times 10^{-19})(1350)(10^{16})} = 1.52 \text{ V/cm}$$

Temperature effect

$$J_0 \equiv \frac{qD_p p_{no}}{L_p} + \frac{qD_n n_{po}}{L_n} \equiv \frac{qD_p n_i^2}{L_p N_D} + \frac{qD_n n_i^2}{L_n N_A}$$

We shall now briefly consider the temperature effect on the saturation current density J_0. We shall consider only the first term in Eq. 64, since the second term will behave similarly to the first one. For the one-sided p^+-n abrupt junction (with donor concentration N_D), $p_{no} \gg n_{po}$, the second term can also be neglected. The quantities n_i, D_p, p_{no}, and L_p $(\equiv \sqrt{D_p \tau_p})$ are all temperature-dependent. If D_p/τ_p is proportional to T^γ, where γ is a constant, then

$$J_0 \approx \frac{qD_p p_{no}}{L_p} \approx q\sqrt{\frac{D_p}{\tau_p}}\frac{n_i^2}{N_D} \propto T^{\gamma/2}\left[T^3 \exp\left(-\frac{E_g}{kT}\right)\right]$$
$$\propto T^{(3+\gamma/2)} \exp\left(-\frac{E_g}{kT}\right) \quad . \qquad (65)$$

The temperature dependence of the term $T^{(3+\gamma/2)}$ is not important compared with the exponential term. The slope of a plot J_0 versus $1/T$ is determined mainly by the energy gap E_g. It is expected that in the reverse direction, where $|J_R| \approx J_0$, the current will increase approximately as $\exp(-E_g/kT)$ with temperature; and in the forward direction, where $J_F \approx J_0 \exp(qV/kT)$, the current will increase approximately as $\exp[-(E_g - qV)/kT]$.

The "Short" Diode

In the previous section, we assumed that both p and n regions were long compared with the minority carrier diffusion lengths. In many pn junction structures, one region may be short compared with the minority carrier diffusion length.

One of the n and p region is shorter than minority carrier diffusion length

$$W_n << L_p$$

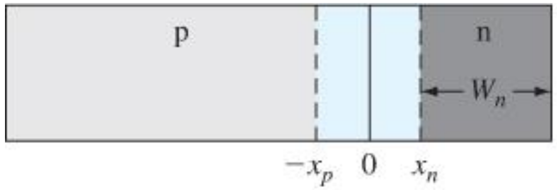


Figure 8.11 | Geometry of a "short" diode.

Steady-state excess minority carrier hole concentration in the n region :

$$\frac{d^2(\delta p_n)}{dx^2} - \frac{\delta p_n}{L_p^2} = 0 \qquad (x > x_n) \Longrightarrow \delta p_n(x) = p_n(x) - p_{n0} = Ae^{x/L_p} + Be^{-x/L_p} \qquad (x \geq x_n)$$

$$\Longrightarrow \delta p_n(x) = p_{n0}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right] \frac{\sinh\left[(x_n + W_n - x)/L_p\right]}{\sinh\left[W_n/L_p\right]}$$

$$\begin{cases} p_n(x_n) = p_{n0}\exp\left(\frac{eV_a}{kT}\right) \\ p_n(x = x_n + W_n) = p_{n0} \end{cases}$$

Boundary conditions

$$\Longrightarrow \delta p_n(x) = p_{n0}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]\left(\frac{x_n + W_n - x}{W_n}\right)$$

$$\begin{cases} \sinh\left(\frac{x_n + W_n - x}{L_p}\right) \approx \left(\frac{x_n + W_n - x}{L_p}\right) \\ \sinh\left(\frac{W_n}{L_p}\right) \approx \left(\frac{W_n}{L_p}\right) \end{cases}$$

Minority carrier hole diffusion current density is :

$$J_p = -eD_p \frac{d(\delta p_n(x))}{dx} = J_p(x) = \frac{eD_p p_{n0}}{W_n}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

- Larger current for short diode

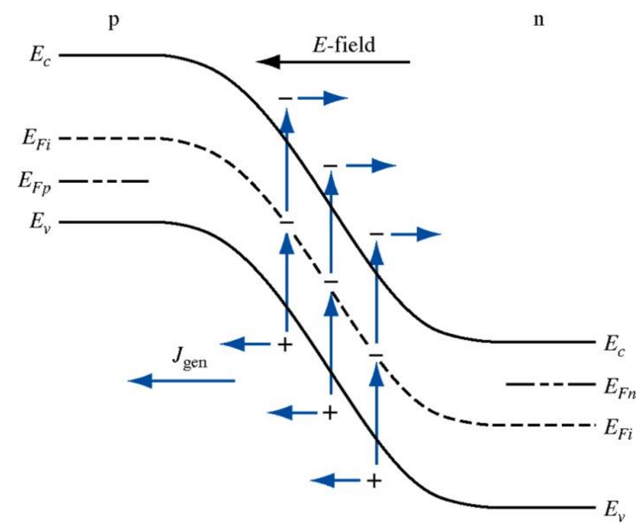
- Constant current → no recombination of minority carriers in the short n region.

The recombination rate of excess electrons and holes, given by the Shockley-Read-Hall recombination theory, was written as

$$R_n = R_p = \frac{C_n C_p N_t (np - n_i^2)}{C_n(n+n') + C_p(p+p')} \equiv R = \frac{\delta n}{\tau} \qquad R = \frac{-C_n C_p N_t n_i^2}{C_n n' + C_p p'}$$

Reverse-biased Generation Current

For a pn junction under reverse bias, mobile electrons and holes are swept out of depletion region. (within the depletion region, $n \approx p \approx 0$)
→ Generation of electron and hole is needed to reestablish thermal equilibrium.



$$J_{\text{gen}} = \frac{e n_i W}{2\tau_0} \qquad J_R = J_s + J_{\text{gen}}$$

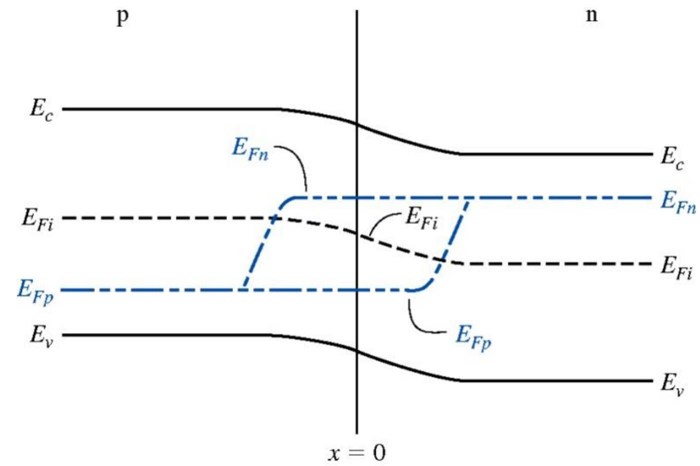
J_S : independent of the reverse bias voltage
J_{gen} : dependent on the depletion width.

The ideal reverse-saturation current density is independent of the reverse-biased voltage. However, J_{gen} is a function of the depletion width W, which in turn is a function of the reverse-biased voltage.

$$R = \frac{-n_i}{\frac{1}{N_t C_p} + \frac{1}{N_t C_n}} \qquad R = \frac{-n_i}{\tau_{p0} + \tau_{n0}} \qquad \tau_0 = \frac{\tau_{p0} + \tau_{n0}}{2} \qquad R = \frac{-n_i}{2\tau_0} \equiv -G \qquad J_{\text{gen}} = \int_0^W eG\,dx$$

Forward bias Recombination Current

Under forward bias, some excess carriers injected across the depletion region exist.
→ Some of these electrons and holes will recombine within the space charge region and not become part of the minority carrier distribution.



$$J_{\text{rec}} = \frac{eWn_i}{2\tau_0} \exp\left(\frac{eV_a}{2kT}\right) = J_{r0} \exp\left(\frac{eV_a}{2kT}\right)$$

$$R = \frac{C_n C_p N_t (np - n_i^2)}{C_n(n + n') + C_p(p + p')}$$

$$n = n_i \exp\left[\frac{E_{Fn} - E_{Fi}}{kT}\right] \qquad p = n_i \exp\left[\frac{E_{Fi} - E_{Fp}}{kT}\right]$$

$$R = \frac{np - n_i^2}{\tau_{p0}(n + n') + \tau_{n0}(p + p')}$$

$$(E_{Fn} - E_{Fi}) + (E_{Fi} - E_{Fp}) = eV_a$$

At the center, $$E_{Fn} - E_{Fi} = E_{Fi} - E_{Fp} = \frac{eV_a}{2}$$

$$R_{\max} = \frac{n_i \;[\exp(eV_a/kT) - 1]}{2\tau_0\;[\exp(eV_a/2kT) + 1]}$$

$$n = n_i \exp\left(\frac{eV_a}{2kT}\right) \qquad p = n_i \exp\left(\frac{eV_a}{2kT}\right)$$

$$J_{\text{rec}} = \int_0^W eR\,dx$$

Total Forward bias Current

The total forward bias current density is the sum of the recombination and the ideal diffusion current densities :

$$J = J_{\text{rec}} + J_D$$

J_{rec} : recombination current density
J_D : ideal diffusion current density.

Because of recombination, additional holes from the *p* side must be injected into the space charge region to establish the minority carrier hole concentration in the *n* region.

$$J_D = J_s \exp\left(\frac{eV_a}{kT}\right)$$

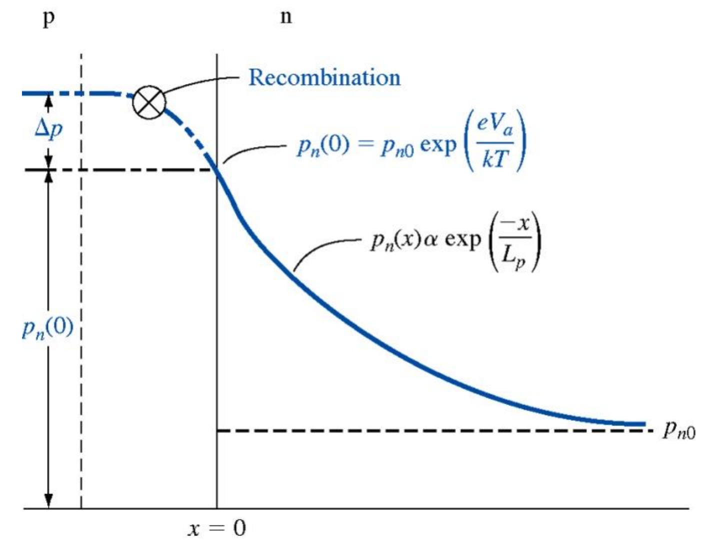
J_S : ideal reverse saturation current density

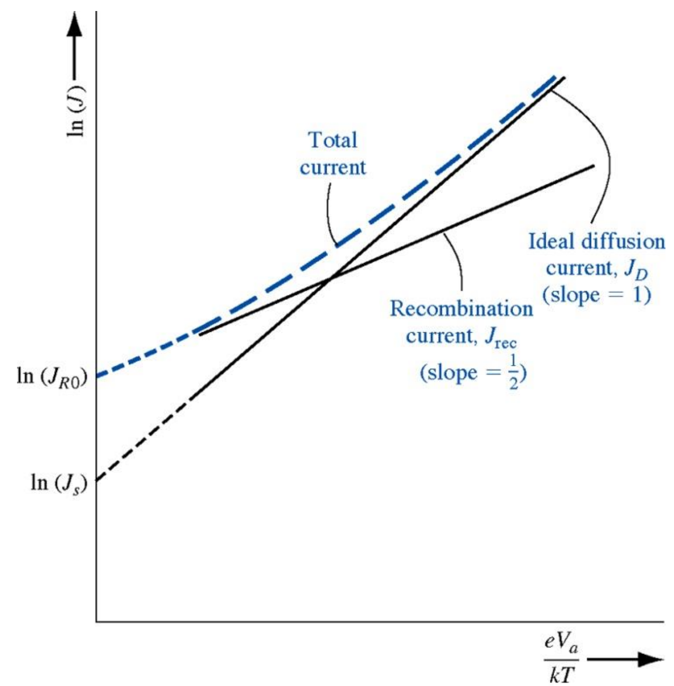
$$\ln J_{\text{rec}} = \ln J_{r0} + \frac{eV_a}{2kT} = \ln J_{r0} + \frac{V_a}{2V_t}$$

$$\ln J_D = \ln J_s + \frac{eV_a}{kT} = \ln J_s + \frac{V_a}{V_t}$$

$$\Longrightarrow \quad I = I_s\left[\exp\left(\frac{eV_a}{nkT}\right) - 1\right]$$

n ≒ 1 at diffusion dominates
n ≒. 2 at low forward bias





High-level injection

Low level injection implies that the excess minority carrier concentrations are always much less than the majority carrier concentration.
However, as the forward-bias voltage increases, the excess carrier concentrations increase and may become comparable or even greater than the majority carrier concentration.

$$n = n_i \exp\left[\frac{E_{Fn} - E_{Fi}}{kT}\right] \qquad p = n_i \exp\left[\frac{E_{Fi} - E_{Fp}}{kT}\right]$$

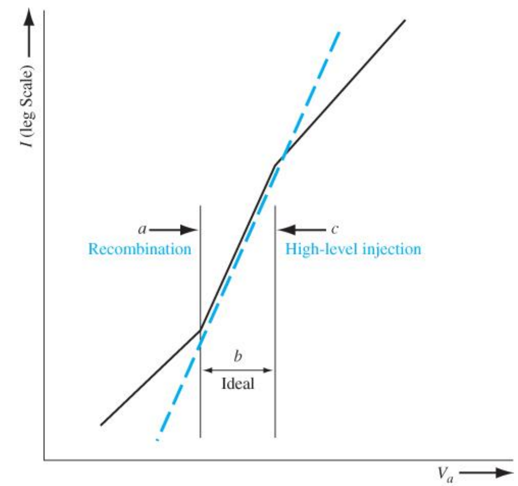
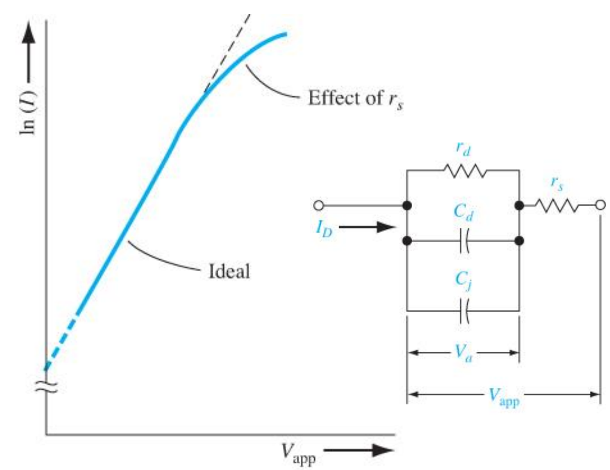
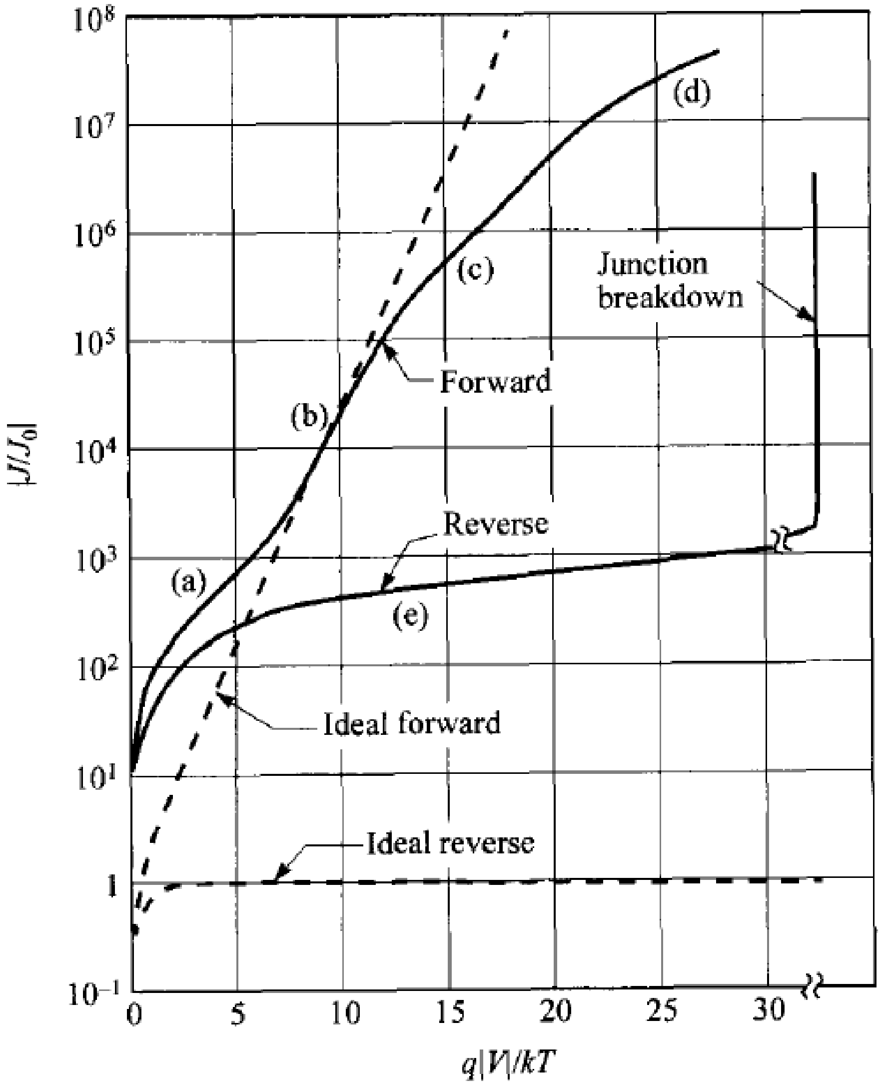


Figure 8.17 | Forward-bias current versus voltage from low forward bias to high forward bias.

Series-resistance

The neutral n and p regions have finite resistances so the actual pn junction will include a series resistance.
A larger applied voltage is required to achieve the same current value when a series resistance is included.





Current-voltage characteristics of a practical Si diode.

(a) Generation-recombination current region.

(b) Diffusion-current region

(c) High-injection region

(d) Series-resistance effect

(e) Reverse leakage current due to generation-recombination and surface effects.

Small-signal model of the pn junction

We have been considering the dc characteristics of the pn junction diode. When semiconductor devices with pn junctions are used in linear amplifier circuits, for example, sinusoidal signals are superimposed on the dc currents and voltages, so that the small-signal characteristics of the pn junction become important.

Diffusion Resistance

$$I_D = I_s \left[\exp\left(\frac{eV_a}{kT} \right) - 1 \right]$$

I_D : Diode current
I_s : diode reverse-saturation current

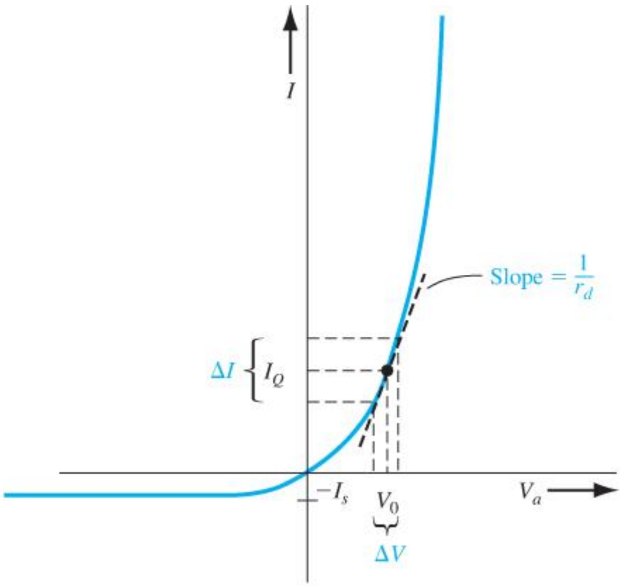


Figure 8.18 | Curve showing the concept of the small-signal diffusion resistance.

The ratio of sinusoidal current to sinusoidal voltage is called the incremental conductance. The small-signal incremental conductance is just the slope of the dc current-voltage curve.
If we assume that the diode is biased sufficiently far in the forward-bias region, then the (-1) term can be neglected and the incremental conductance becomes,

$$g_d = \left. \frac{dI_D}{dV_a} \right|_{V_a = V_0} = \left(\frac{e}{kT} \right) I_s \exp\left(\frac{eV_0}{kT} \right) \approx \frac{I_{DQ}}{V_t}$$

Then, the small-signal incremental resistance is the reciprocal function,

$$r_d = \left. \frac{dV_a}{dI_D} \right|_{I_D = I_{DQ}} = \frac{V_t}{I_{DQ}}$$

The incremental resistance is also known as the diffusion resistance.

Small-signal model of the pn junction

Diffusion capacitance

The mechanism of charging and discharging of holes in the n region and electrons in the p region leads to a capacitance. This capacitance is called diffusion capacitance.
The physical mechanism of this diffusion capacitance is different from that of the junction capacitance. We show that the magnitude of the diffusion capacitance in a forward-biased pn junction is usually substantially larger than the junction capacitance.

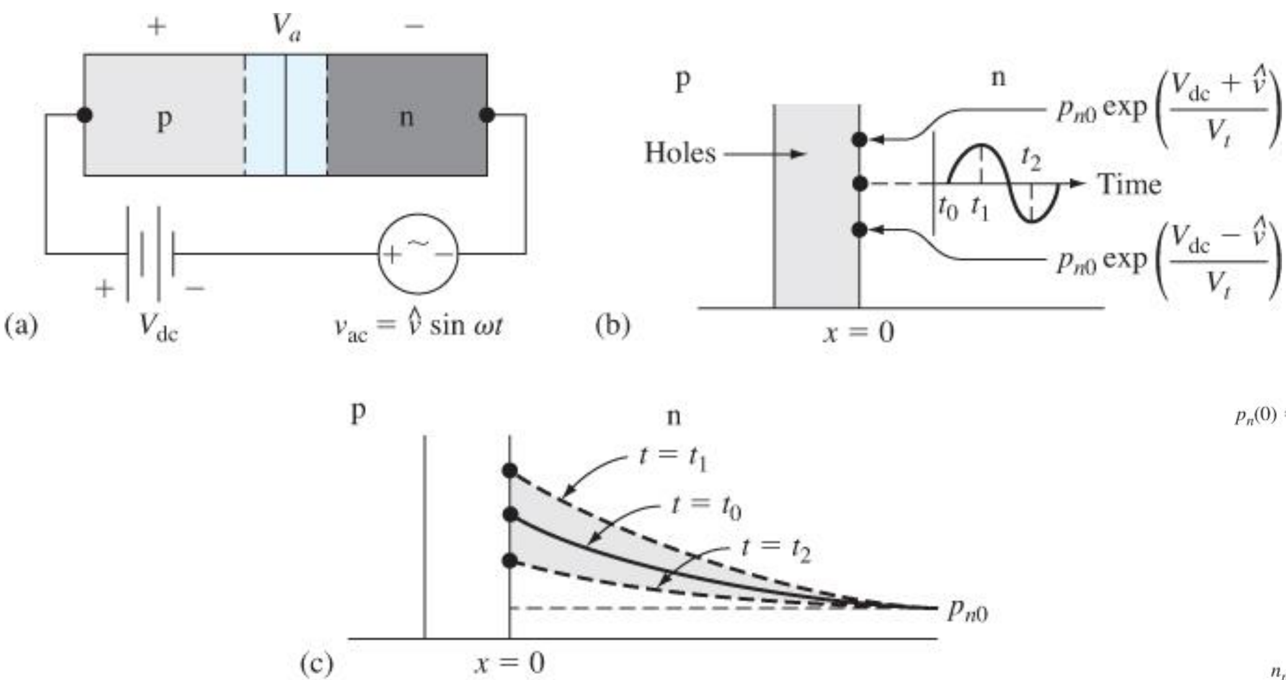


Figure 8.19 | (a) A pn junction with an ac voltage superimposed on a forward-biased dc value; (b) the hole concentration versus time at the space charge edge; (c) the hole concentration versus distance in the n region at three different times.

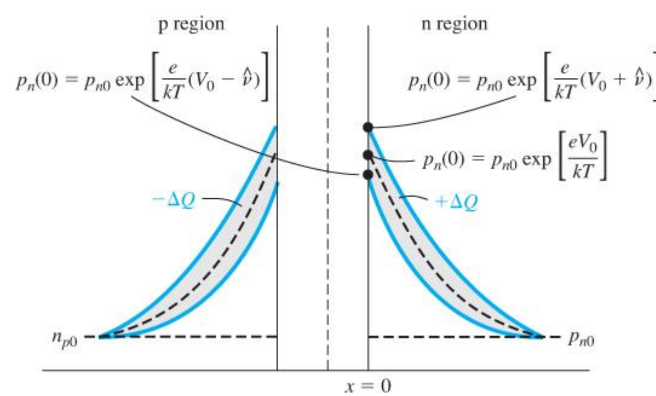


Figure 8.21 | Minority carrier concentration changes with changing forward-bias voltage.

Transient Analysis of Diode

The speed of the pn junction diode in switching state is determined by the short transient time between "on" and "off" states.

$$I = I_F = \frac{V_F - V_a}{R_F}$$

$$I = -I_R \approx \frac{-V_R}{R_R}$$

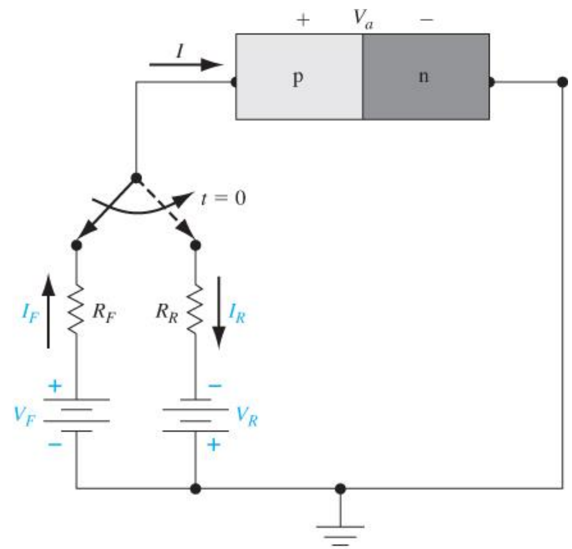


Figure 8.24 | Simple circuit for switching a diode from forward to reverse bias.