

Theory of Semiconductor Devices (반도체 소자 이론)

Lecture 8. The PN junction

Young Min Song

Associate Professor

School of Electrical Engineering and Computer Science Gwangju Institute of Science and Technology

http://www.gist-foel.net

ymsong@gist.ac.kr, ymsong81@gmail.com

A207, 232655



PN Junction

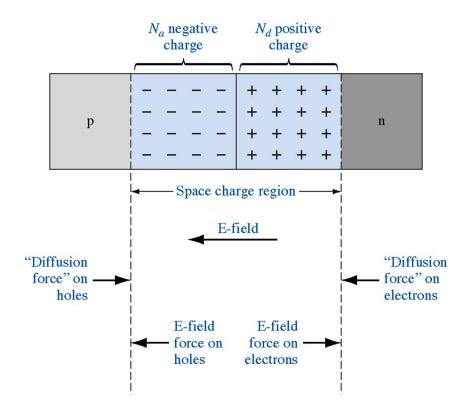
Basic structure in most semiconductors

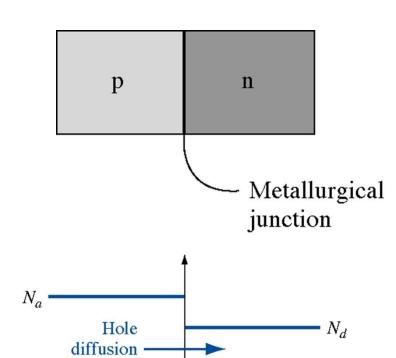
Metallurgical junction

Step junction (abrupt junction)

Space charge region (depletion region)

Diffusion force vs. Electric field





x = 0Essentially all electrons and holes are swept out of the space charge region by the electric field.

Electron diffusion

In thermal equilibrium, the diffusion force and the E-field force exactly balance each other.



n

Two assumption:

Boltzmann approximation is valid. Each semiconductor region is nondegenerately doped.

Complete ionization exists. The temperature of the pn junction is not too low.

Built-in Potential Barrier

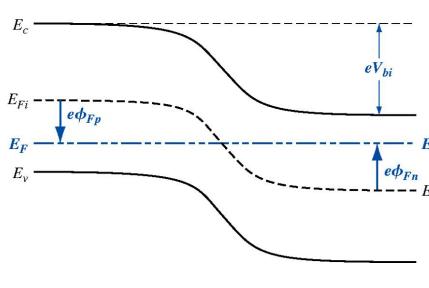
: prevent diffusion flow

$$V_{bi} = |\phi_{Fn}| + |\phi_{Fp}|$$

$$n_0 = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right] = n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right]$$

$$= n_i \exp\left[\frac{-(e\phi_{Fn})}{kT}\right] \qquad e\phi_{Fn} = E_{Fi} - E_F$$

$$\implies \phi_{Fn} = \frac{-kT}{e} \ln\left(\frac{N_d}{n_i}\right)$$



$$p_0 = N_a = n_i \exp\left[\frac{E_{Fi} - E_F}{kT}\right]$$

$$e\phi_{Fp} = E_{Fi} - E_F$$

$$\phi_{Fp} = +\frac{kT}{e} \ln\left(\frac{N_a}{n_i}\right)$$

$$V_{bi} = \frac{kT}{e} \ln \left(\frac{N_a N_d}{n_i^2} \right) = V_t \ln \left(\frac{N_a N_d}{n_i^2} \right)$$

p



 ρ (C/cm³)

Calculation of E-field

$$\frac{d^2\phi(x)}{dx^2} = \frac{-\rho(x)}{\epsilon_x} = -\frac{dE(x)}{dx}$$

 $\phi(x)$: electric potential E(x): electric field $\rho(x)$:

$$\rho(x) = -eN_a \qquad -x_p < x < 0$$

$$\rho(x) = -eN_a \qquad -x_p < x < 0$$

$$\rho(x) = eN_d \qquad 0 < x < x_n$$

$$E = \int \frac{\rho(x)}{\epsilon_s} dx = -\int \frac{eN_a}{\epsilon_s} dx = \frac{-eN_a}{\epsilon_s} x + C_1$$

$$E = \int \frac{(eN_d)}{\epsilon_s} dx = \frac{eN_d}{\epsilon_s} x + C_2$$

$$\rho(x) = eN_d \qquad 0 < x < x_n$$

$$\mathbf{E} = \int \frac{(eN_d)}{\epsilon_s} \, dx = \frac{eN_d}{\epsilon_s} x + C_2$$

$$E = \frac{-eN_d}{\epsilon_s}(x + x_p) \qquad -x_p \le x \le 0$$

$$E = \frac{-eN_d}{\epsilon_s}(x_n - x) \qquad 0 \le x \le x_n$$

$$N_a x_p = N_d x_n$$

$$e^{-eN_d}$$

$$0 \le x \le x_n$$

$$N_a x_p = N_d x_p$$

$$\phi(x) = -\int E(x) dx = \int \frac{eN_a}{\epsilon_s} (x + x_p) dx$$

$$\phi(x) = \frac{eN_a}{\epsilon_s} \left(\frac{x^2}{2} + x_p \cdot x \right) + C_1'$$

$$\phi(x) = \int \frac{eN_d}{\epsilon_s} (x_n - x) dx$$

$$\phi(x) = \frac{eN_d}{\epsilon_s} \left(x_n \cdot x - \frac{x^2}{2} \right) + C_2'$$

$$\phi(x) = \frac{eN_a}{\epsilon_s} \left(\frac{x^2}{2} + x_p \cdot x \right) + C_1'$$

$$\phi(x) = \frac{eN_d}{\epsilon_s} \left(x_n \cdot x - \frac{x^2}{2} \right) + C_2'$$

$$C'_{1} = \frac{eN_{a}}{2\epsilon_{s}} x_{p}^{2}$$

$$\phi(x) = \frac{eN_{a}}{2\epsilon_{s}} (x + x_{p})^{2} \quad (-x_{p} \le x \le 0)$$

$$C'_{2} = \frac{eN_{a}}{2\epsilon_{s}} x_{p}^{2}$$

$$\phi(x) = \frac{eN_{d}}{\epsilon_{s}} \left(x_{n} \cdot x - \frac{x^{2}}{2}\right) + \frac{eN_{a}}{2\epsilon_{s}} x_{p}^{2} \quad (0 \le x \le x_{n})$$

$$V_{bi} = |\phi(x = x_{n})| = \frac{e}{2\epsilon_{s}} \left(N_{d}x_{n}^{2} + N_{d}x_{p}^{2}\right)$$

$$|x_n| = \frac{e}{2c} \left(N_d x_n^2 + N_d x_p^2 \right)$$

$$V_{bi} = |\phi(x = x_n)| = \frac{e}{2\epsilon_s} \left(N_d x_n^2 + N_a x_p^2 \right)$$

Space Charge Width

Distance that the space charge region extends from the metallurgical junction

$$N_{a}x_{p} = N_{d}x_{n} \implies x_{p} = \frac{N_{d}x_{n}}{N_{a}}$$

$$V_{bi} = |\phi(x = x_{n})| = \frac{e}{2\epsilon_{s}} \left(N_{d}x_{n}^{2} + N_{a}x_{p}^{2}\right)$$

$$x_{n} = \left\{\frac{2\epsilon_{s}V_{bi}}{e} \left[\frac{N_{a}}{N_{d}}\right] \left[\frac{1}{N_{a} + N_{d}}\right]\right\}^{1/2}$$

$$x_{p} = \left\{\frac{2\epsilon_{s}V_{bi}}{e} \left[\frac{N_{d}}{N_{a}}\right] \left[\frac{1}{N_{a} + N_{d}}\right]\right\}^{1/2}$$

$$W = x_{n} + x_{p} \quad W = \left\{\frac{2\epsilon_{s}V_{bi}}{e} \left[\frac{N_{a} + N_{d}}{N_{a}N_{d}}\right]\right\}^{1/2}$$

Example 7.2: Consider silicon pn junction at T = 300K with $N_a = 10^{16}$ cm⁻³ and $N_d = 10^{15}$ cm⁻³

$$W = \left\{ \frac{2\epsilon_s V_{bi}}{e} \left[\frac{N_a + N_d}{N_a N_d} \right] \right\}^{1/2}$$

$$= \left\{ \frac{2(11.7)(8.85 \times 10^{-14})(0.635)}{1.6 \times 10^{-19}} \left[\frac{10^{16} + 10^{15}}{(10^{16})(10^{15})} \right] \right\}^{1/2}$$

$$= 0.951 \times 10^{-4} \text{ cm} = 0.951 \ \mu\text{m}$$

$$E_{\text{max}} = \frac{-eN_d x_n}{\epsilon_s} = \frac{-(1.6 \times 10^{-19})(10^{15})(0.864 \times 10^{-4})}{(11.7)(8.85 \times 10^{-14})} = -1.34 \times 10^4 \text{ V/cm}$$

Reverse Bias : a positive voltage is applied to the n region with respect to the p region.

→ Fermi level is no longer constant through the system. (quasi-Fermi level!)

$$V_{ ext{total}} = |\phi_{Fn}| + |\phi_{Fp}| + V_R$$
 $V_{ ext{total}} = V_{bi} + V_R$

The electric fields in the neutral p and n regions are essentially zero, or at least very small!!

$$W = \left\{ \frac{2\epsilon_s(V_{bi} + V_R)}{e} \left[\frac{N_a + N_d}{N_a N_d} \right] \right\}^{1/2}$$

Example 7.2 : A silicon pn junction at T = 300K with $N_a = 10^{16}$ cm⁻³ and $N_d = 10^{15}$ cm⁻³. Let $V_R = 5$ V.

$$W = \left\{ \frac{2(11.7)(8.85 \times 10^{-14})(0.635 + 5)}{1.6 \times 10^{-19}} \left[\frac{10^{16} + 10^{15}}{(10^{16})(10^{15})} \right] \right\}^{1/2}$$

$$W = 2.83 \times 10^{-4} \text{ cm} = 2.83 \ \mu\text{m}$$

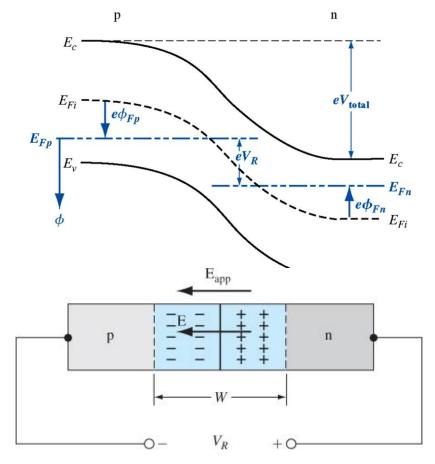


Figure 7.8 | A pn junction, with an applied reverse-biased voltage, showing the directions of the electric field induced by V_R and the space charge electric field.



Junction Capacitance

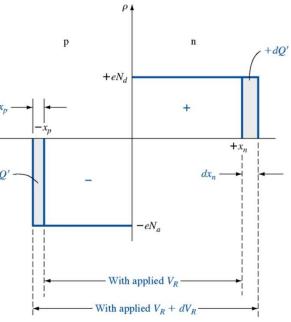
Separated positive and negative charges across the depletion region. \rightarrow Capacitance!!

$$C' = \frac{dQ'}{dV_R}$$
 where $\frac{dQ' = eN_d dx_n = eN_a dx_p}{Q': \text{C/cm}^2, \ C': \text{F/cm}^2}$

$$C' = \frac{dQ'}{dV_R} = eN_d \frac{dx_n}{dV_R} \Longrightarrow C' = \left\{ \frac{e\epsilon_s N_a N_d}{2(V_{bi} + V_R)(N_a + N_d)} \right\}^{1/2} = \frac{\epsilon_s}{W}$$

$$x_n = \left\{ \frac{2\epsilon_s(V_{bi} + V_R)}{e} \left[\frac{N_a}{N_d} \right] \left[\frac{1}{N_a + N_d} \right] \right\}^{1/2}$$
 same as using x_p



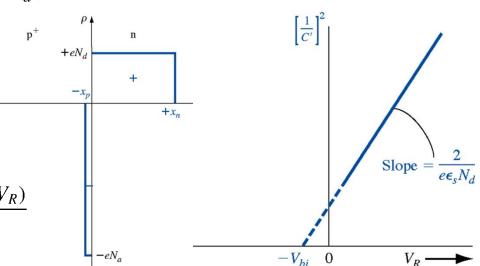


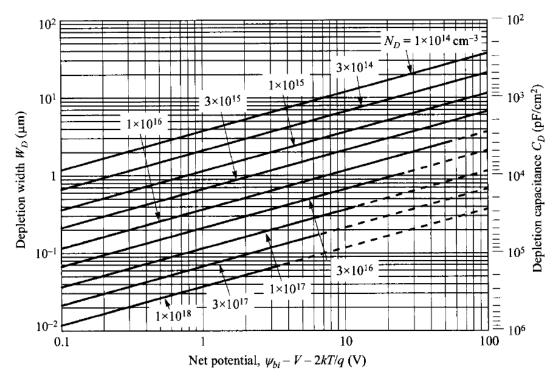
One-sided Junction (p^+n junction for $N_a \gg N_d$)

$$W pprox \left\{ rac{2\epsilon_s(V_{bi} + V_R)}{eN_d}
ight\}^{1/2}$$

$$x_p \ll x_n \quad W \approx x_n$$

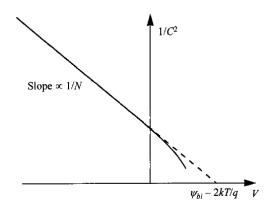
$$C' pprox \left\{ rac{e\epsilon_s N_d}{2(V_{bi} + V_R)}
ight\}^{1/2} \implies \left(rac{1}{C'}
ight)^2 = rac{2(V_{bi} + V_R)}{e\epsilon_s N_d}$$





Depletion-layer width and depletion-layer capacitance per unit area as a function of net potential for one-sided abrupt junctions in Si. Doping N is from the lightly doped side. Dashed lines represent breakdown conditions.

imation. A more accurate result for the depletion-layer properties can be obtained by considering the majority-carrier contribution in addition to the impurity concentration in the Poisson equation, that is, $\rho \approx -q[N_A-p(x)]$ on the p-side and $\rho \approx q[N_D-n(x)]$ on the n-side. The depletion width is essentially the same as given by Eq. 19, except that ψ_{bi} is replaced by $(\psi_{bi}-2kT/q)$.* The correction factor 2kT/q comes about because of the two majority-carrier distribution tails^{5,6} (electrons in n-side and holes in p-side, as shown by the dashed lines in Fig. 1a) near the edges of the depletion region. Each contributes a correction factor kT/q. The depletion-layer width at thermal equilibrium for a one-sided abrupt junction becomes



$$W_D = \sqrt{\frac{2\,\varepsilon_s}{qN} \left(\psi_{bi} - \frac{2\,k\,T}{q}\right)} \,. \tag{23}$$

The semiconductor potential and the capacitance-voltage data are insensitive to changes in the doping profiles that occur in a distance less than a Debye length.

The Debye length L_D is a characteristic length for semiconductors and is defined as

$$L_D \equiv \sqrt{\frac{\varepsilon_s kT}{q^2 N}} = \sqrt{\frac{\varepsilon_s}{qN\beta_{th}}}.$$

This Debye length gives an idea of the limit of the potential change in response to an abrupt change in the doping profile. Consider a case where the doping has a small increase ΔN_D in the background of N_D , the change of potential near the step is given by

$$\begin{split} n &= N_D \mathrm{exp} \Big(\frac{\Delta \psi_i q}{kT} \Big), \\ \frac{d^2 \Delta \psi_i}{dx^2} &= -\frac{q}{\varepsilon_s} (N_D + \Delta N_D - n) = -\frac{q N_D}{\varepsilon_s} \bigg[1 + \frac{\Delta N_D}{N_D} - \mathrm{exp} \Big(\frac{\Delta \psi_i q}{kT} \Big) \bigg] \\ &\approx -\frac{q N_D}{\varepsilon_s} \bigg[1 + \frac{\Delta N_D}{N_D} - \Big(1 + \frac{\Delta \psi_i q}{kT} \Big) \bigg] \approx \frac{q^2 N_D}{\varepsilon_s kT} \Delta \psi_i \end{split}$$

If the doping profile changes abruptly in a scale less than the Debye length, this variation has no effect and cannot be resolved, and that if the depletion width is smaller than the Debye length, the analysis using the Poisson equation is no longer valid.

At thermal equilibrium the depletion-layer widths of abrupt junctions are about 8L_D for Si, and 10L_D for GaAs.

For a doping density of 1016 cm-3, the Debye length is 40 nm.

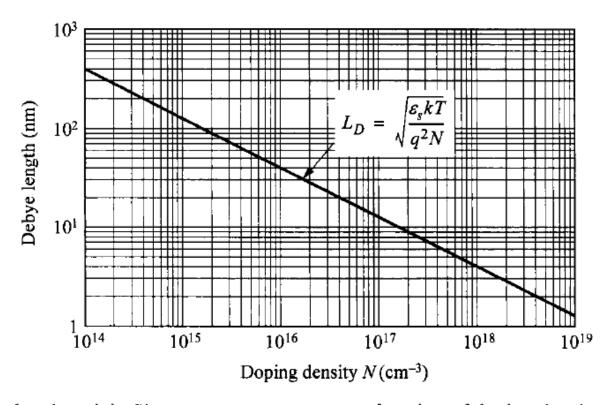


Fig. 4 Debye length in Si at room temperature as a function of doping density N.



Zenor vs. Avalanche Breakdown

Breakdown: An abrupt increase of the reverse-bias current. (Permanent damage or NOT).

Zenor: In highly doped junction through a tunneling mechanism. In a highly doped junction, the conduction and valence bands on opposite sides of the junction are sufficiently close during reverse bias that electrons may tunnel directly from the valence band on the p side into the conduction band on the n side.

Avalanche: Impact ionization in SCR. The process occur when electrons and/or holes acquire sufficient energy to create electron-hole pairs by colliding with atomic electrons within the depletion region. For most pn junctions, the predominant breakdown mechanism will be the avalanche effect.

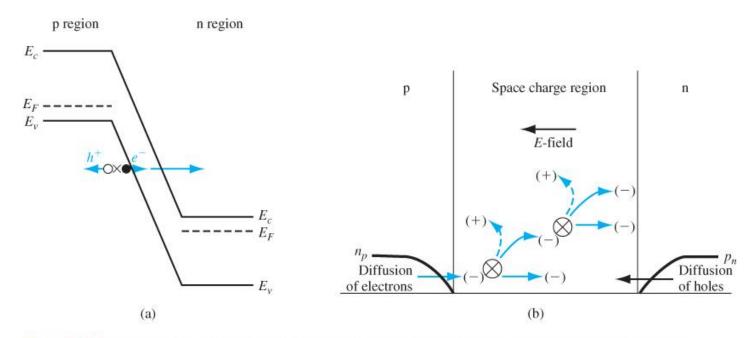


Figure 7.12 I (a) Zener breakdown mechanism in a reverse-biased pn junction; (b) avalanche breakdown process in a reverse-biased pn junction.



Linearly Graded Junctions

Junction made by diffusion can be regarded as linearly graded junction in the metallurgical point.

$$\rho(x) = eax$$

$$\frac{dE}{dx} = \frac{\rho(x)}{\epsilon_s} = \frac{eax}{\epsilon_s}$$

$$E = \int \frac{eax}{\epsilon_s} dx = \frac{ea}{2\epsilon_s} \left(x^2 - x_0^2 \right) \quad \leftarrow E(-x_0) = E(+x_0) = 0$$

$$\phi(x) = -\int E dx = \frac{-ea}{2\epsilon_s} \left(\frac{x^3}{3} - x_0^2 x \right) + \frac{ea}{3\epsilon_s} x_0^3 \quad \leftarrow \phi(-x_0) = 0$$

$$\phi(x_0) = \frac{2}{3} \cdot \frac{eax_0^3}{\epsilon_s} = V_{bi}$$

For reverse biased junction,

$$x_0 = \left\{ \frac{3}{2} \cdot \frac{\epsilon_s}{ea} (V_{bi} + V_R) \right\}^{1/3} \qquad \xrightarrow[-dQ']{}$$

$$C' = \frac{dQ'}{dV_R} = (eax_0) \frac{dx_0}{dV_R}$$

$$= \left\{ \frac{ea\epsilon_s^2}{12(V_{bi} + V_R)} \right\}^{1/3}$$

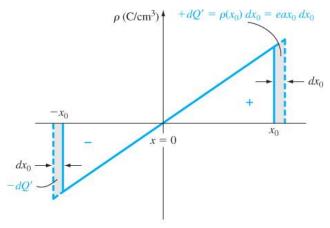


Figure 7.18 | Differential change in space charge width with a differential change in reverse-biased voltage for a linearly graded pn junction.

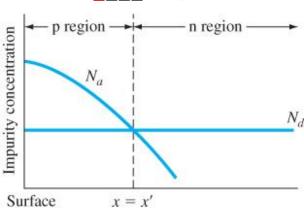


Figure 7.16 | Impurity concentrations of a pn junction with a nonuniformly doped p region.

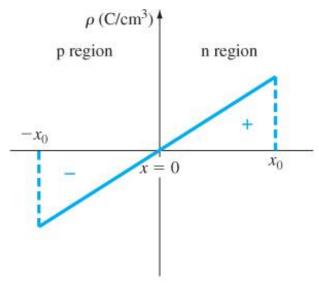


Figure 7.17 | Space charge density in a linearly graded pn junction.